



Generating Functions for ${}_pR_q$ Polynomial

Yogesh M. Thakkar^{1,2} · Ajay K. Shukla¹

Accepted: 17 March 2023

© The Author(s), under exclusive licence to Springer Nature India Private Limited 2023

Abstract

In this paper, we define ${}_pR_q$ polynomial, which is denoted by $\mathcal{R}_n(z)$. We also discuss some generating functions and recurrence relations for $\mathcal{R}_n(z)$ polynomial and its applications.

Keywords Gamma function · Generalized hypergeometric polynomial · Generalized Sister Celine's polynomial · Generalized Rice's polynomial · Bateman's polynomial · Hermite polynomial · Leguerre polynomial · Gegenbauer polynomial · Jacobi polynomial and Legendre polynomial

Mathematics Subject Classification 33E12 · 33B15 · 33C20 · 33C45 · 33D99

Introduction and Preliminaries

The generalized hypergeometric function [2, 3, 15] with p numerator and q denominator parameters is defined as,

$${}_pF_q \left(\begin{matrix} \mathbf{a_p} \\ \mathbf{b_q} \end{matrix} \middle| z \right) = \sum_{k=0}^{\infty} \frac{(\mathbf{a_p})_k}{(\mathbf{b_q})_k} \frac{z^k}{k!}, \quad (1)$$

where $p, q \in \mathbb{Z}^+ \cup \{0\}$, $z \in \mathbb{C}$, $\operatorname{Re}(\mathbf{a_p}) > 0$, $\operatorname{Re}(\mathbf{b_q}) > 0$. Here $\mathbf{a_p}$ stands for the set of p parameters a_1, a_2, \dots, a_p , $\mathbf{b_q}$ stands for the set of q parameters b_1, b_2, \dots, b_q , $(\mathbf{a_p})_k = \prod_{i=1}^p (a_i)_k$, $(\mathbf{b_q})_k = \prod_{j=1}^q (b_j)_k$ and $(a)_k$ is a Pochhammer symbol [15] defined by

$$(a)_k := \frac{\Gamma(a+k)}{\Gamma(a)} = \begin{cases} 1 & (k=0; a \in \mathbb{C} \setminus \{0\}) \\ a(a+1) \dots (a+k-1) & (k \in \mathbb{N}; a \in \mathbb{C}). \end{cases} \quad (2)$$

✉ Ajay K. Shukla
ajayshukla2@rediffmail.com; aks@amhd.svnit.ac.in

Yogesh M. Thakkar
yogeshthakkar1979@gmail.com; yogeshthakkar@somaiya.edu

¹ Department of Mathematics and Humanities, Sardar Vallabhbhai National Institute of Technology, Surat, Gujarat 395 007, India

² Department of Science and Humanities, K. J. Somaiya College of Engineering, Somaiya Vidyavihar University, Mumbai 400077, India