Some formulas for the function $R_3\left[\mu,\mu',\delta,\delta';\gamma;\upsilon,\tau,z_1,z_2\right]$

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Abstract

In this paper, we obtain confluence formulas, series representations, integral representations and differentiation formulas for Appell-type extension of the ${}_pR_q(\alpha,\beta;z)$ function.

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1 Introduction and preliminaries

Recently, we discussed eleven functions of two variables [7], in which four functions are Appell-type extension of the ${}_pR_q(\nu,\tau;z)$ function and remaining seven functions are confluent functions of Appell-type extension of the ${}_pR_q(\nu,\tau;z)$ function as mentioned below.

These functions are defined for $\mu,\mu',\delta,\delta',\gamma,\gamma',\upsilon,\tau,z_1,z_2\in C,\gamma,\gamma'\notin Z^-\cup\{0\}$, $Re\left(\upsilon\right)\geq 0, Re\left(\tau\right)>0$ and $\left|z_1\right|,\left|z_2\right|<\infty$.

$$R_1\left[\mu,\delta,\delta';\gamma;\upsilon,\tau,z_1,z_2\right]=\sum_{p\geq 0}\sum_{q\geq 0}\frac{1}{\Gamma\left(\upsilon\left(p+q\right)+\tau\right)}\frac{(\mu)_{p+q}(\delta)_p(\delta')_q}{(\gamma)_{p+q}}\frac{{z_1}^p}{p!}\frac{{z_2}^q}{q!}\tag{1.1}$$

$$R_2\left[\mu,\delta,\delta';\gamma,\gamma';\upsilon,\tau,z_1,z_2\right]=\sum_{p\geq 0}\sum_{q\geq 0}\frac{1}{\Gamma\left(\upsilon\left(p+q\right)+\tau\right)}\frac{(\mu)_{p+q}(\delta)_p(\delta')_q}{(\gamma)_p(\gamma')_q}\frac{{z_1}^p}{p!}\frac{{z_2}^q}{q!}\tag{1.2}$$

$$R_3\left[\mu,\mu',\delta,\delta';\gamma;\upsilon,\tau,z_1,z_2\right]=\sum_{p\geq 0}\sum_{q\geq 0}\frac{1}{\Gamma\left(\upsilon\left(p+q\right)+\tau\right)}\frac{(\mu)_p(\mu')_q(\delta)_p(\delta')_q}{(\gamma)_{p+q}}\frac{{z_1}^p}{p!}\frac{{z_2}^q}{q!}\tag{1.3}$$

$$R_4\left[\mu,\delta;\gamma,\gamma';\upsilon,\tau,z_1,z_2\right]=\sum_{p\geq 0}\sum_{q\geq 0}\frac{1}{\Gamma\left(\upsilon\left(p+q\right)+\tau\right)}\frac{(\mu)_{p+q}(\delta)_{p+q}}{(\gamma)_p(\gamma')_q}\frac{{z_1}^p}{p!}\frac{{z_2}^q}{q!}\tag{1.4}$$

$$R\Phi_1\left[\mu,\delta;\gamma;\upsilon,\tau,z_1,z_2\right]=\sum_{p\geq 0}\sum_{q\geq 0}\frac{1}{\Gamma\left(\upsilon\left(p+q\right)+\tau\right)}\frac{(\mu)_{p+q}(\delta)_p}{(\gamma)_{p+q}}\frac{{z_1}^p}{p!}\frac{{z_2}^q}{q!}\tag{1.5}$$

$$R\Phi_2\left[\delta,\delta';\gamma;\upsilon,\tau,z_1,z_2\right]=\sum_{p\geq 0}\sum_{q\geq 0}\frac{1}{\Gamma\left(\upsilon\left(p+q\right)+\tau\right)}\frac{(\delta)_p(\delta')_q}{(\gamma)_{p+q}}\frac{{z_1}^p}{p!}\frac{{z_2}^q}{q!}\tag{1.6}$$

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