National Initiative on Mathematics Education (NIME 2011-12)

National Conference on Mathematics Education

January 20-22, 2012
Mumbai, India

Proceedings

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Preface

This volume brings together the papers, and abstracts of invited talks and posters, presented in the National Conference on Mathematics Education held at the Homi Bhabha Centre for Science Education, Tata Institute of Fundamental Research, Mumbai from January 20-22, 2012. The National Conference is a part of the National Initiative in Mathematics Education (NIME 2011-2012).

Mathematics is regarded as a core part of the school curriculum across the world. In higher education, it is part of the core curriculum for a wide range of basic and professional courses. However, it is also widely perceived as a difficult subject, to be tolerated rather than learned and appreciated. As a school subject it is a major hurdle in passing the final qualifying examination. Many teachers, educators and mathematicians have responded to the challenge of teaching mathematics effectively in diverse ways: some have developed powerful pedagogical approaches or learning materials, some have introduced innovations in their teaching, some have worked with teachers, some with students, and some have taken up research to understand more deeply the teaching and learning of mathematics.

The National Initiative on Mathematics Education (NIME 2011-12) is aimed at taking account of these initiatives to prepare a report on the status and outlook of mathematics education in India at all levels of education – elementary, secondary and tertiary. Under the NIME initiative five regional conferences on mathematics education have been held across the country. The series of conferences concluded with the National Conference at HBCSE, the proceedings of which are presented in this volume. These conferences were aimed at providing a broad and representative platform for initiatives in mathematics education at various levels. The proceedings of these conferences will form inputs to the National Report on Mathematics Education. The report will be a part of the National Presentation by India at the International Congress of Mathematics Education (ICME-12) to be held in Korea in July 2012.

Submissions were invited for the National Conference in the form of papers and posters on the following themes:

1. Historical and Cultural aspects of mathematics and mathematics education
2. Systemic and policy aspects of education
3. Curriculum and pedagogy at various levels
4. Teacher education and development

The articles and abstracts included in this volume address issues related to all the themes mentioned above. Together with the proceedings of the regional conferences, they provide the most comprehensive picture available so far of issues and challenges in mathematics education as well as efforts made to address them. However, this is only an initial step, and much more concerted effort is needed to raise the quality and reach of mathematics education at all levels in the country.

Meena Kharatmal and Aaloka Kanhere have worked hard to prepare this compilation of papers and abstracts. Manoj Nair has worked at express speed and brought all his skills to bear in producing an attractive layout for the proceedings and organizing the printing. Arindam Bose, Shikha Takker and Jeenath Rahaman and Saritha V.N. have helped in editing the citations and references. I thank all of them.

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Abstracts of Invited Talks

Invited Extended Talks

Re-visioning School Mathematics Curriculum to Address Social Justice Concerns in India

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It is well acknowledged within the community of educationists and in particular those working in mathematics education that a multitude of differences – a finely graded class and caste differences and gender difference within them, regional differences ranging from metropolitan cities to the remotest locations in rural India, linguistic differences and segregation along religion to mention a few – operate in the Indian context and if 'education for all' should mean more than a mere slogan, then we need to identify ways in which this ideal can be realized. The challenge lies not just in the range and the complex combinations of differences but in the fact that these differences are hierarchical and that education tends to privilege the already privileged. Mathematics as a school subject acquires a special position because of its perceived importance, and its role as an effective gatekeeper for those who belong to the social margins. The focus group paper on mathematics lists gender as one of the systemic issues that needs to be addressed specifically in mathematics, as 'mathematics tends to be regarded as a "male domain" ' and cites instances of invisibility of the female gender. It also acknowledges that 'caste based discrimination manifests' in certain forms in the context of mathematics education and 'nearly half the children drop out of school during the elementary stage'. When viewed alongside the national success in Olympiads, its reputation for picking and training the 'talented' in the premier institutions of the country, its place in academic mathematics in the international arena, the aggressive way in which middle class promotes learning of mathematics in the expensive tutorial centers outside the school, the dismal performance of government school children in basic numeracy tests and primary school mathematics amply illustrates whose interests mathematics education serves and who are disposable.

There has been a lot of work on gender and mathematics education, particularly in the West. In the process of studying how gender figures in mathematics education, it has become clear that a range of factors operate to minimize opportunity for girls to succeed in mathematics and this includes notions about gender difference in inherent ability, absence of role models, conscious and unconscious bias operating on the part of the teachers and co-students, learners' self concept about ability to do mathematics, classroom dynamics, curriculum and textbook content and so on. Similarly, there is a lot of literature on social justice issues in mathematics education that problematize long standing notions about, what is mathematics and why teach mathematics. Some of the questions that have been raised are "Are there alternate ways to organize mathematics education to serve the interests of all the learners?" "Could mathematics education contribute to building a democratic society and critical citizenship?"

There is very little literature on how gender and other social categories operate in mathematics education in the Indian context. However, based on the experiences of organizations that have worked at the grass root level, it is known that for a large number of children from socio-economically disadvantaged section who fail to attain even minimum levels of learning in mathematics at the primary school level, the upper primary and secondary school mathematics would function to keep them from accessing benefits of higher education. The presentation would raise some of these issues and suggest some possible approaches to mathematics curriculum design.

School Mathematics, the Discipline of Mathematics and Transitions

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What constitutes the discipline of mathematics is best described by the dictum: "mathematics is as mathematicians do". But then should whatever mathematicians do necessarily have an influence on mathematics as taught in school ? A priori, there is no reason to: the goals of school mathematics and that of the discipline of mathematics are very different. Teaching the discipline in school is not so much for producing competence and expertise in that discipline as for enriching the resources of the child as well as for meeting social and developmental goals.

It can therefore be argued that school mathematics should be taught in whatever way that suits it best, and processes essential to the practice of mathematicians need not be relevant. De facto, this is one reason why school mathematics concentrates on using mathematics rather than producing mathematics of any kind at all. Roughly, this is analogous to urging children to "practise your notes" because you cannot even appreciate music properly without doing so, but it is hard (or boring) to do so unless one wants to become a musician.

This debate would be entirely academic were it not for its import on curricular choices, and transitions. A curriculum that aims at producing a certain level of mathematical competence (say in calculus) by the time the student exits the system is often tower-shaped, each layer of competence built on preceding ones.

On the other hand, mathematics as a mode of intellectual enquiry might well be given an architecture that's broader, "closer to the ground". Indeed, such curricular choices may be seen in the way some countries have gone about in shaping their curricula.

The other aspect referred to above is that of transitions. Perhaps the most dramatic example of this is in the way the concepts of infinity and continuity are treated in school and in the university. But this is also true of the transition from arithmetic to algebra and from algebra and geometry to trigonometry in school. These transitions often lack grounding and motivation from the student's viewpoint because they focus on skill building (for essential use later) and not on mathematical processes that necessitate them.

We present an argument that doing mathematics in school is desirable from such a perspective, and that it will only help universalisation: the goal of engaging every child in the mathematics classroom with a sense of success.

Undergraduate Mathematics Education

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Undergraduate education is akin to a pivot or keystone that holds together myriad strands that contribute to society. It is the case that whether it is a teacher at school or at the University, a researcher or a manager, or anyone holding a white-collar job, the one common aspect they share is that of having received undergraduate education.

The degree at the undergraduate level paves the way for the future. It is usually carefully chosen keeping in mind interests, aptitude and career opportunities ahead. It comprises those years in one's life where one is moving from a system governed by `restrictions' to one that is about `making choices and decisions' that stay for the rest of one's life.

Mathematics is explicitly and implicitly present in many things of importance to society. Mathematics has a role to play in so many different fields: innovations in medicine, digital encryption, communication technology, modelling real life phenomena, predicting disasters, organisation of enterprises, business and transport to name a few.

Yet for a layperson there is not only a lack of awareness about the indispensability of mathematics but instead there is a marked tendency to `ignore' mathematics. Indeed, it is fashionable to acknowledge publically the deep-seated fear of the subject that stems from memories of bad experiences with mathematics usually encountered at school.

It is important for mathematicians and mathematics educators to acknowledge and seek ways of changing this. It is neither good for the discipline nor society to be in a situation where possibly over 50% can only recount a dislike and unhappiness associated with mathematics.

At the heart of mathematics education lies undergraduate mathematics education. It would be impossible to tackle any of the problems associated with mathematics education, at any level without intervention at the undergraduate level. After all, the harbingers of change, if there are to be any, will be the teachers, policy makers and the creators and dispensers of curriculum. And without fail, each one of them will have been shaped by their undergraduate mathematics education.

Given the vital importance that undergraduate education occupies, it is necessary that we examine the tenets that govern undergraduate mathematics education in our country.

What institutions or courses comprise undergraduate mathematics education? What should be the aims and goals of undergraduate mathematics education? What is the state of undergraduate mathematics education in our country? Are our courses geared to meeting the stated goals and aims?

While these and many other questions occur naturally when one reflects on mathematics education at the undergraduate level, the talk will focus on the issues of curriculum design, pedagogy and the quality of instruction, training of faculty. These will be examined primarily through the lens of an Honours degree in mathematics.

Another important but neglected area is the interaction between mathematics/ mathematicians/ mathematics educators involved in higher education and those that are involved with school mathematics education. The talk will also briefly dwell on what can and should be done to foster interactions.

Transforming the Elementary Mathematics Curriculum: Issues and Challenges

Anita Rampal
Faculty of Education, Delhi University

The National Curriculum Framework 2005 and the Right to Education Act 2009 have reframed the vision for elementary education, within which the mathematics curriculum has a major enabling role to play. In addition to the shift in focus from 'narrow' to 'higher' goals of mathematical learning, from content to mathematical learning environments, from procedural knowledge to processes, the NCF has also called for a curriculum that removes the widely prevalent sense of math anxiety while engaging every learner with a sense of success. Such restructuring within a socio-cultural perspective of learning further challenges deeply entrenched notions of school math, and locates the curriculum in the contested arena of knowledge, power and social justice. This presentation delineates the process of enabling a transformative curriculum for elementary mathematics, through analyses of policy documents, syllabi and textbooks, and classroom observations.

Maths Education – The Efforts from Outside the System

Hriday Kant Dewan
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India has a large educational system which is now largely State governed. The present system of public education has become very large and has expanded in the years since independence to reach all parts of the country. Since independence the government has systematically created institutional structure at different levels to ensure that all children have access to some school. There have been discussions and efforts to make processes of education for children more meaningful and these include efforts in addition to mathematics. There are structures that have been set up to help and support the teacher in the school, to prepare appropriate materials for children and teachers and to build capacity of the teacher to fulfill their role.

Continuing from a time when large proportion of schools and colleges were outside the Govt. institutional framework, we still have a large number of institutions that are run by organisations not in the Govt. Framework. Apart from this in the last four decades through interaction with the Govt. Structures they have attempted to influence it. It can be said that their efforts have also supported and influenced the government institutional framework.

The paper discusses the contribution of non-government institutions including maths teachers association and individuals to the efforts of reaching quality education in maths to all children. We use the examples of the work of a few organisations from each category and point out the need for supporting such efforts, the constraints that they face and the need to encourage them to expand and attempting to build stronger networks of these institutions with structures set up in the public system.

The paper argues that the present education system controlled by the government at the Central and State levels has a lot of gaps and needs to be supported by other processes. Education in a democratic polity is the concern of the people and they must have a role in it. The paper argues that for educational discourse to be rich and inclusive it must have a diversity of models. There must be also a variety of explorations to seek resolution of the complex issues of ensuring everyone learns mathematics.

The paper points out that given the nature of the question and its multiple dimensions there are many aspects to be explored and the need for small intervention to examine the possibilities of ideas becoming pragmatically possible. It argues that the participation of NGOs and individuals help build a discourse in the Society, fermenting of ideas and a larger participation and engagement among those thinking about education. The paper points out that while there have been many initiatives of this kind and have included wide set of areas there is not sufficient documentation of these. There is also inadequate analysis of these efforts and while the learnings from these efforts are not utilized widely, their short-comings and gaps in their implementation are also not analysed and corrected. The lack of linkages between large institutions and structures and these efforts also reduces the likelihood of researching such efforts.

The paper also argues that the efforts have been restricted to only a few dimensions and are not widespread. Many regions have seen none. Most of the efforts have focused around junior classes and have aimed to include similar ideas in maths education. These ideas also need to be examined. It is important to think about ways of promoting more such efforts, enabling the enlargement of the scope and encouraging the possibility of other organisations exploring interventions. It is also important that organisations build their capacity to influence the larger system and have the opportunity to explore working with institutions that will help them build their capacity.

Mathematics Education Research across the World

K. Subramaniam
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In the last two decades, mathematics education research has rapidly spread across many countries and has acquired an international character. In contrast perhaps to research in the natural sciences or mathematics, in mathematics education research diversification has grown with internationalization. Indeed, diversity of educational systems, approaches and cultures is itself a major research focus. This is because mathematics education research has a practical focus; its ultimate goal is to improve the mathematical education of students and of citizens. Since education is of human beings and is realized through socially organized systems, mathematics education research has a large overlap with the social sciences.

Thus a characteristic, even definitive feature of mathematics education research is its situatedness in particular contexts and its connection with questions of policy, planning and practice. Mathematics education research is the systematic, rigorous, critical and reflective study, using the methods of scientific inquiry, of the problems of mathematics education in specific contexts and their possible resolution. It is not something that is more necessary in countries that achieve a poor mathematical education of students, or in countries that achieve the highest levels of mathematics education. Mathematics education research is necessary and actively pursued in a diversity of countries and contexts. To cite a couple of examples, a large and active community of researchers exists in South Korea, which is a small country with a high level of achievement in mathematics education and in South Africa, which does not have a high level of achievement and has an entirely different socio-political context.

A driving force behind the internationalization is the organization of the mathematics education fraternity at the global level led by the International Commission for Mathematics Instruction (ICMI), which is a part of the International Mathematical Union (IMU). The ICMI inherits the organizational reach and the culture of the IMU, and incorporates an ethos that respects and encourages diversity. The ICME Congresses that it organizes is arguably the most important international forum in mathematics education. Research is one of the central foci of the ICMI and is organized in a variety of ways – through the Topic Study Groups and survey teams in the ICME Congress, the ICMI Studies commissioned from time to time, and the affiliate study groups of which the oldest are the International Group for the Psychology of Mathematics Education (PME) and the International Study Group on the Relations between the History and Pedagogy of Mathematics (HPM). The PME in particular has its own large reach through its annual conferences.

The pattern of research submissions to major conferences such as the ICME or PME gives an indication of popular themes of research. For the ICME-12 Congress, there are 37 Topic Study Groups, of which the five that received the most number of submissions in order are (i) in-service professional development of math teachers (ii) maths education in a multilingual and multicultural environment (iii) pre-service maths education of teachers (iv) research on classroom practice and (v) analysis of uses of technology in the teaching of math s. Importance of an issue is only one of the factors contributing to the popularity of a TSG. Tractability of research and closeness to practical engagement of researchers are others. Teacher education and development which received the most submissions, is not only important, but also close to the work of many maths education researchers who are located in departments or colleges that run teacher education programmes. The low popularity of some TSGs is not an indication of low importance. The TSG on the teaching and learning of measurement for example, is a critical but under-researched area that received far fewer submissions. In my talk, I'll briefly discuss the important issues in a few core areas of research and the kinds of methods adopted to study these.

A second important reason for the internationalization of mathematics education research is the role of international comparisons of learning achievement, which are themselves an outcome of the process of globalization. The most prominent of these are the TIMSS (Trends in International Mathematics and Science Study) and PISA (Programme for International Student Achievement), which are quite different from each other in terms of what they aim to test. Low National performances have frequently galvanised policy changes and initiatives within countries, while the overall results have spurred much research in mathematics education worldwide. Some countries, especially the East Asian countries of Singapore, South Korea, Hong Kong, Japan, China and Taiwan, have consistently performed at the top in all these comparisons. Some other countries such as the Netherlands, Belgium and Australia have also faired well consistently. Some countries have done better in TIMSS than in PISA (Russia, Hungary, USA), while Finland which is one of the top performers in PISA has rarely participated in TIMSS.

The consistent top performances of East Asian countries and the gap between these and other countries has prompted many researchers to study the underlying reasons. I will briefly cite a couple of research programmes that have yielded important insights, one from the West and one from the East. The TIMSS 1995 and 1999 studies included a video study component led by researchers from the U.S., where mathematics lessons from 8 countries, both top-performing as well as low-performing, were video recorded and carefully analysed. Stigler and Hiebert, who have written extensively on the findings from these studies, point out that there is much diversity in the teaching approaches and methods even within the high achieving countries, suggesting that there multiple "best" approaches to teaching mathematics. However, common characteristics of the mathematics teaching in the high achieving countries are that they "(1) attend to important mathematical (conceptual) relationships and (2) involve (engage) students in doing serious mathematical work" (Stigler and Hiebert, 2009, p. 35), a finding that the authors stress is consistent with "independent research on mathematics teaching and learning over the past 75 years". One of the outcomes of these studies is the close attention now being paid to the pre-service and in-service education of teachers and the realization that deep knowledge of content, albeit of a sort specialized to the requirement of teaching, is required even to teach at the elementary level.

A set of studies from East Asian researchers have pointed to the complex array of possible factors that may play a role in the high performance of their countries, including the value placed on learning in their cultures (see for example, Leung & Li, 2010). They point out that some countries have been influenced by developments in the West, while some are relatively insulated from these developments. These studies also portray a dynamic picture of change and development across East Asian countries with regard to the school mathematics curriculum and its aims, the pedagogical techniques, and the organization of teacher preparation. The studies taken together suggest that there is much that is to be learned about how to design and achieve a high level of mathematics education for the vast majority of citizens of the world.

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History of Indian Mathematics and its Implications for Mathematics Education

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Towards the end of his scholarly preface to a recently published work on History of Indian Mathematics, David Mumford succinctly captures the present state of awareness of history: Too many people still think that mathematics was born in Greece and more or less slumbered until the Renaissance. The state described by Mumford is not so much because of the lack of information about the contributions of the other civilizations, as much as it is due to the fact that it remained confined among the specialists. Consequently, the general picture that emerged from the books on history of mathematics with regard to the Indian contributions till recent times were quite incomplete and at times misleading.

Having said this as preamble, I would like to straightaway address the question, what purpose does the study of History of Indian mathematics, in particular the techniques for solving certain class of problems (with plenty of illustrative examples drawn from day to day life) described in the Indian mathematical texts would serve in mathematics education?

- The first and foremost, it would make the mathematics education complete. Besides providing the background in which various concepts got developed, it would also provide a multi-cultural perspective which is presently lacking in the educational curricula.
- Secondly, the Indian approach being algorithmic, it would enable the students to have a sense of a different flavor of mathematics. For example, the prescriptions given in Sulbasutra texts for finding surds, the recurrence relation given by Aryabhata for evaluating sine function, the techniques for solving indeterminate equations given by Brahmagupta and Bhaskara, the method of arriving at infinite series for trigonometric functions as well as fast convergent approximations to the 'Gregory-Leibniz' series for π given by the Kerala mathematicians (two centuries earlier) are quite exciting and unique. Some of these techniques along with several illustrative examples and demonstrations provided in the texts, could perhaps prove to be simpler for the students in assimilating the concepts with much ease.
- Thirdly, it would help in a major way in slowly eradicating the false pictures that have already been created by some of the accounts — be it generated by Eurocentric bias or Indi-centric bias.
- Lastly, making the students aware of the major achievements of their own ancestors— particularly in their impressionable age — is quite likely to boost their self-confidence and also provide the necessary motivation in building a self-reliant nation.

It may not be out of context to recall the commitment on the part of China as a nation to produce hundreds of volumes and thousands of research papers in the past few decades highlighting the history of traditional science and technology in China. It is believed that this serious enterprise, not only to produce but also to disseminate this knowledge among the society, could have played a key role in the academic repositioning of China among the top few in the global scenario today.

Finally, unless we expose the young bright minds with the significant contributions of their own civilization, it would be next to impossible to excite and attract at least a handful of them to work in the area of History of

Indian Mathematics (which of course is closely wedded to Astronomy) that needs to be more deeply explored for having a broader picture of almost three millennium long history.

Invited Talks

Missed Opportunities in the Higher Secondary Mathematics Curriculum

Shailesh A Shirali
Community Math Centre, Rishi Valley School, Rishi Valley, Andhra Pradesh

The position paper of the Mathematics Focus Group of NCF 2005 puts forth a beautifully articulated vision of what mathematics at school level should be, gives an analysis of the situation in India (the status of mathematics teaching at the primary, high school and higher secondary levels), considers questions of assessment and teacher preparation, and goes on to make a few key recommendations:

1. Shifting the focus of mathematics education from achieving narrow goals to higher goals;
2. Engaging every student with a sense of success, at the same time offering conceptual challenges to the emerging mathematician;
3. Changing modes of assessment to examine students' mathematisation abilities rather than procedural knowledge;
4. Enriching teachers with a variety of mathematical resources.

The 'higher goals' mentioned in (1) call for a shift in focus in teaching-learning: now the *processes* of mathematics begin to occupy centre stage, as distinct from topic content and algorithms. (What gets tested in examinations is mostly the mastery of algorithms.) Some of these processes are:

- Formal problem solving
- Use of heuristics
- Use of patterns
- Estimation and approximation
- Optimization
- Visualisation
- Representation
- Reasoning and proof
- Making connections
- Mathematical communication

Over the past five years some part of (1) has been realized in the textbooks published by NCERT, chiefly at the primary and upper primary levels, but to some extent at the high school and higher secondary levels too; and some part of (3) has been realized in the changing style of assessment, chiefly by the CBSE, which has made the 10^{th} standard examination optional. But we still have a long way to go.

In (2) and (4), very little has been achieved (particularly (4), teacher preparation).

In this talk I look briefly at the CBSE higher secondary math curriculum. I take up one strand of the NCF 2005 document, related to item (1) – the question of depth and breadth of coverage of topics – and list what I consider to be 'wasted effort' and 'missed opportunities' in this curriculum; 'wasted' in the sense that the time spent on these topics yields very little, and the time could certainly have been spent much more usefully on other topics; 'missed opportunity' in the sense that within the same listing of topics, with no change of syllabus required, much more can be done but is not.

Contribution of Teacher's Associations towards Mathematics Education

M. Mahadevan
Association of Mathematics Teachers of India, Chennai

The Associations of Teachers of Mathematics are said to be functioning in various places. In some states more than one association is functioning. The objectives and actions of such associations vary. In some cases they run journals, publish books and occasionally organize meets to celebrate important days. This draft is a little more detailed on the origin and way of functioning of our association, The Association of Mathematics Teachers of India with head quarters in Chennai.

The origin of the association was in 1964 with the publication of a journal *The Mathematics Teacher*. The Association was formally registered in 1965.

The major objectives, brief notes on important functionaries and some of the activities will be included in the paper and given as a small hand book to the participants. The activities include conducting of National Mathematics Talents Competitions from class 5 to class 12 twice as per our norms and once for seniors at the undergraduate level.

We also conduct orientation programmes for raising talents related to Olympiad leading to IMO, workshops for teachers locally and at their places on demand to impress upon them the innovative strategies to be tried during class room interaction attending to all levels of performers and also help the talented in and /or outside the class informally to enable them keep up their talents.

Our national conferences conducted in various parts of the country are a great attraction for teachers and students discussing pedagogical perspectives and giving confidence building paper presentations by teachers and students. Exhibition of teaching aids, endowment lectures also are included therein besides Quiz related to general knowledge in mathematics.

Distinguished mathematics teacher award is also given to deserving teachers to enthuse others to work with interest and enthusiasm.

We are happy to note that the dream of one of our founder members Sri P.K.Srinivasan, to declare the birth day of Srinivasa Ramanujan as resolved in our 38th conference at Rajahmundry as National Mathematics Day has now been declared so by our Prime Minister as seen by us in the media when we were in the 46th conference. We suggest ideas and they take time to fruition. It may be possible for this dream of ours- To enable children to learn Mathematics with interest and enthusiasm- will also be realized by our continuous efforts.

We look forward to understanding cooperation from one and all of you directly or indirectly.

Distance is not a Barrier

Parvin Sinclair
NCERT, Delhi

The Open University system was initiated in India with APOU, and in 1985 began the Indira Gandhi National National Open University, IGNOU. At the school level, the National Open School started in 1989, offering courses at the secondary and senior secondary levels.

All these systems have taught mathematics in the distance mode, using the print medium largely,with weekly contact programmes. In this paper, I will be presenting the strengths and weaknesses of the mathematics teaching and learning processes in the Open and Distance Learning institutions.

Teaching Calculus

E. Krishnan
Kerala Mathematics Teachers Association, Thiruvananthapuram

Apart from the division of mathematics education quantitatively into various grades classes, there is also a qualitative division, based on the concepts introduced at various stages. The transition from whole numbers to fractions and that from actual numbers to their symbolic representation in algebra are some examples of this. A major turning point of this type, usually first encountered in Class 11 and developed further in the University is the study of Calculus. Most textbooks trivialize the study of this subject as just another bag of computational tricks, ignoring its profound conceptual and pedagogical implications. This talk gives an outline of how calculus can be taught in a more meaningful manner.

In this scheme, the notion of limits underlying calculus has to be gradually developed, starting from high school classes. The approximations of certain fractions and also irrationals in terms of finite decimals can be the starting points of introduction to the idea of limits. Computation of curvilinear areas and volumes can serve as intuitive introduction to the techniques of integral calculus. The emphasis in teaching analytical geometry should be on the translation between algebraic properties of functions and the geometric properties of its graphs and should include examples from physics also. With this preparation, differentiation can be introduced in Class 11, indicating the history of its origin as an attempt to study rectilinear non-uniform motion with uniform acceleration and then to the study of general rates of change. Graphs can be used to show how the same technique can be used in analytic geometry to compute the slopes of tangents. This naturally leads to the introduction of limits of general functions which can be geometrically introduced using their graphs. Algebraic techniques can then be introduced to compute limits, and later derivatives, of combinations of functions. Integration is often introduced as the inverse of differentiation, without any motivation. Instead, it is to be started independently, starting with the computation of curvilinear areas and the various attempts in history to tackle this problem. The fact that this problem can be solved by the reversal of differentiation, with which it has no apparent connection, is to be the climax of this story of intellectual efforts across geography and chronology.

Math Education: Can we explore and exploit ICT?

Sujata Ramdorai
Tata Institute of Fundamental Research, Mumbai

There is a new approach to inclusive education that hopes to exploit technology and social networking. We shall talk about this theme and its relevance to math education in India.

Mathematics Education Research in India: Issues and Challenges

Rakhi Banerjee
Ambedkar University, Delhi

Mathematics education acquires many connotations: from popularizing mathematics, making some esoteric mathematics accessible to common people to more engaged and systematic studies of issues related to teaching and learning of mathematics at various levels. This presentation would try to bring together some of these various forms of initiatives/ research traditions and critically try to understand the present status of mathematics education in the country.

The earliest of the initiatives, like Eklavya's Khushi-Khushi series of books, where mathematics is taught by integrating it with other subjects at the primary level, or Digantar's hands-on approach for teaching mathematics, were largely 'intuitive' and based on an understanding of what teaching and learning of mathematics must entail. A major concern at that time, not just in India but across the world, was the high number of students failing in mathematics. One of the reasons cited for this was the meaninglessness and abstractness of mathematical symbols and operations. These initiatives made an attempt to change teaching of mathematics in certain ways so that it becomes possible to make sense of mathematics by connecting it with experiences in the world. Several such initiatives, including NCERT's NCF-2005, have been taken since then. And all of them made certain assumptions while designing the teaching interventions or curricular interventions. I would try to explicate some of them and bring to light the dilemmas and the struggles we constantly face in our efforts to change the teaching and learning of mathematics.

One of the challenges in front of us is to undertake serious and systematic studies on issues related to curriculum development and implementation, teacher practice, teacher development, student learning in different environments, etc., using research methods appropriate for such complex issues. There does not exist many (or any) systematic study of what actually is the impact of following certain practices/ assumptions in schools across the country – either on students or on teachers, beyond our impressions that the alternative methods work well with students. Unfortunately, the large numbers of university departments of education have not been able to establish traditions of doing subject and content specific research and have also not been able to provide a platform and model for good education of pre-service teachers. Barring some individual, institutional and group efforts, we have not been able to systematically carry out in-service teacher education also. It seems a large number of us are still caught in Piagetian understanding of stage-wise development of children.

In a nutshell, we do not have sufficient research base for either understanding students' learning of mathematics or teachers' practice of teaching of mathematics. This of course has many implications for policy formulation as well. We need better theoretical grounding to understand the very complex nature of the problems that we face in education, and in specific mathematics education, as well as empirical support to back up our policy formulations.

We are also well aware of the power that mathematics as a discipline provides to those who are able to master its language. Thus, we do see a variety of efforts to this end make students capable of understanding and using mathematics in challenging contexts. I will try to elucidate a few of these.

Panel Discussion

Assessment Culture and its Impact on Mathematics Education

Amber Habib
Shiv Nadar University, New Delhi

Higher education in India is guarded by a host of exams, the general exams run by the national/state education boards as well as the entrance tests held by various specialized institutes or coalitions of institutes. In recent years there has been much discussion and tinkering with the school curriculum, but it has not yet significantly affected the nature of these tests. We will look at the ways in which these tests can affect the achievement of the aims of curriculum reform, their impact on the mathematical thinking of students entering higher education, and the ways which institutes of higher education are adopting to counter their influence.

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The right place of assessment in learning: Educators and society use assessment as a tool in three different ways: as a device to measure how much a student has learned, as an instrument to motivate learning and as a filter to allocate resources. One obvious effect of the latter two uses is that assessment becomes an end in itself and learning is thrown by the wayside. The other is that summative assessment is subjecting students to tremendous stress. The presentation will briefly go into the other 'backwash' effects on curriculum of using assessment to motivate and screen students.

The rest of the presentation will explore the role of assessment in learning. This will be based on my teaching experience at Centre For Learning (CFL) for almost two decades. We can say with conviction at CFL that we help students learn and master concepts without resorting to comparative evaluation and continuous testing. Moreover, in their school leaving 'high stakes' exams they are at no disadvantage as compared to students who have been subject to these forms of assessment continually for twelve years or more!

Paper Presentations

Children's Understanding of the Geometrical Concepts: Implications for Teaching Geometry

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Abstract: This article attempts to suggest the implications for teaching-learning based upon the findings of my research work. The article constitutes the part of a learned paper, focused on the epistemological and the cognitive perspectives of geometry and M.Phil. dissertation that investigated the developmental changes in conceptual understanding of the concepts of 'triangle' and 'circle' of the students of grades V and VII. The analysis is based upon the theoretical framework of the Houdement and Kuzniak's geometrical paradigms (2003), Fischbein theory of figural concepts (1993) and the mathematical meaning of 'Triangle' and circle. The findings suggest that there exists a gap between the personal and mathematical meaning of these concepts and there were not many differences found in the understanding of the students of grade V and VII, in spite of beginning of the formal introduction of the geometry in the curriculum from grade VI.

Key words: figural concept, geometrical paradigms, grades V & VII, Triangle and Circle

Introduction

The 'Triangle' (Polygon) and 'Circle' are the geometrical figures that are presented to children at early primary stage. In Indian schools, there are different ways of how these concepts are introduced to children at primary stage where geometry is not considered as the formal subject. In the NCERT mathematics books of primary classes, the triangle is presented by showing three dimensional figures where the two-dimensional representation is triangular in form. However at upper primary level as geometry is introduced as a formal subject, an over-detailed explanation such as: "A triangle is a two-dimensional shape (a polygon) with three sides and three angles" is taught. Some teachers start with polygons, named according to the number of sides; a three-sided shape is named "triangle". Whereas some teachers introduced it by recognizing different shapes and distinguish from other geometrical shapes. By the time children pass primary stage, mathematical object triangle become more sophisticated and properties, area and other concepts related to triangle are introduced assuming that children are well versed with the meaning of triangle as a mathematical object. Circle is informally introduced at class IV. It is usually introduced by showing wheels and bangles followed by explanation, center, radius and diameter and drawing of circle in NCERT text books. The main objective is to familiarize the children with the triangular and circular shapes around them. In Class VI it is again introduced with little bit more explanation as from class VI geometry is formally introduced. But definition of circle is not explained as such. However, students must know the drawing of circle using compass and freehand. They must also know its parts like diameter, radius and center

Vighi (2005) explored the progressive development of the concept of triangle in the minds of primary children. Findings revealed that most groups carried out classifications "incorrect" in Euclidean sense, but nonetheless based on well defined criteria. Many scholars (Duval, 1998; Parzysz, 1988; Berthelot & Salin, 1998) identified several strong difficulties related to the use of figures in solving geometry problems at secondary school concerning both construction of figures and proofs of properties of geometric figures. Fischbein (1993) proposed the notion of 'figural concept' such that, while a geometrical figure such as a square can be described as having intrinsic conceptual properties (in that it is controlled by geometrical theory), it is not solely a concept, it is an image too (*Ibid*, p141). Accordingly, geometrical reasoning is characterized by the interaction between these two aspects, the figural and the conceptual. Fischbein (1993) also suggested that "the process of building figural concepts in the students' mind should not be considered a spontaneous effect of usual geometry courses". Monaghan (2000) discussed the way in which students perceive various quadrilaterals, and particularly the connection between the perception and description of a given geometrical figure.

Researchers have also shown that most of the difficulties are experienced by the students during the transition from elementary/primary to secondary school which is evident in their performance in most topics, especially in mathematics. It is considered to be as a critical life event, since progressing from one level of education to the next, students may experience major changes in school climate, educational practices, and social structures (Rice,2001). Deliyianni, Elia & Gagatsis (2009) investigated the role of various aspects of apprehension, i.e., perceptual, operative and discursive apprehension, in geometrical figure understanding of primary and secondary school student. Findings revealed differences between primary and secondary school students' performance and in the way they behaved during the solution of the tasks. Panaoura & Gagatsis (2009) studied the geometrical reasoning of primary and secondary school students in order to investigate the strategies and common errors students make while transition from natural geometry (the objects of this paradigm of geometry are material objects) to Natural Axiomatic Geometry (definitions and axioms are necessary to create the objects in this paradigm of geometry). These findings stress the need for helping students progressively move from the geometry of observation to the geometry of deduction.

The present study

The basic purpose of the study was to explore the developmental changes in conceptual understanding of the geometrical concepts of 'Triangle' and 'Circle' among children of grades V and VII. It was an attempt to look at perceptual, lexical and figural aspect (Fischbein, 1993) among students of class V and VII and to see the changes from V to VII class students. The study also tried to explore the factors contributing to the development of the concept viz. Pedagogical Practices in mathematics, Support available at home regarding learning mathematics, Resources available at school for mathematics. An attempt was also made to suggest the implications of the findings for teaching of geometry.

Theoretical frameworks

Houdement and Kuzniak's Theoretical Perspective

The article focuses on the part of the theoretical perspective that considers geometry as a theory of space, which tends to represent the local properties of real space. Its more elaborate form is R3 with the structure of a Euclidean space. This perspective divides geometry into three paradigms viz**.** Geometry I (Natural Geometry) which is related to reality. Perception, experiment and deduction are the means to act on the material objects. Geometry II (Natural Axiomatic Geometry) where hypothetical deductive laws in axiomatic systems are to be used for validation , however relation to reality remains important. Geometry III (Formalist Axiomatic Geometry), the system of axioms is complete and independent of its possible applications to the world. The only criterion of truth is consistency (i.e. absence of contradictions). "Fundamental principle is that the various proposed paradigms are homogeneous: it is possible to reason inside one paradigm without knowing the nature of the other" (Houdement & Kuzniak, 2003).

Fischbein's theory of figural concepts

In Fischbein(1993), it is argued that geometrical figures are characterized by both conceptual and sensorial properties. A geometrical figure is a mental abstract, ideal entity, the meaning of which is governed by a definition. At the same time, it is an image: it possesses extensiveness (spatiality), shape and magnitude. In geometrical reasoning the two categories of properties should merge absolutely, with the sensorial components providing the dynamics of invention and the conceptual component guaranteeing the logical course of the mathematical process.

Triangle

What is a triangle? Is a triangle an abstraction of concrete objects? In nature there are very few examples of triangles. If we try to find, in the manner of Galilean philosophy, correspondence between objects in nature and mathematical shapes, it is difficult to place the triangle. According to Plato, triangle is an idea, which exists independently of human thought, and according to Aristotle, a triangle is an immanent shape upon real objects. "In Aristotle's view, in order to speak scientifically about a concept, one needs to have a "universal understanding" of it, that is to know its true nature" (Speranza, 1996). Euclid defines the rectilinear figures (Definition XIX) those bound by straight lines, trilateral shapes are bound by three straight lines. He classifies them according to sides or angles (Definitions XX and XXI).

Circle

The invention of the wheel is a fundamental discovery of properties of a circle. Mathematically, a circle is defined as the set of all points that are at the same distance from a given point, called the center. That distance is called the radius. The word radius comes from the Latin word for rod or spoke of a wheel, and the radii, or spokes, radiate out from the center.

The definition of a circle is the set of points all at the same distance from a given point called the center. This definition can be used to draw a circle in the sand with a piece of string, or on paper with a compass. The word 'center' comes from the Greek word "kentron" meaning sharp point, that of a compass. Each point is equally distant from the axle of the wheel, making the tire roll smoothly. This makes the circle the ideal shape for gears and wheels and anything that rolls. The circle is the same on all sides. Designers choose this shape when they design an object that has no top or bottom, front or back, and can be used equally well from all sides.

Methodology

This study was done on 132 students of the three government schools of the capital (New Delhi) of India in two phases. A pilot study was conducted with the students of grades 4-7 in order to select the appropriate classes for the research as well as to try out activities to be conducted for the research. Classes 5th and 7th were found to be appropriate as class 5th being end of primary mathematics education, students are familiar with most geometrical concepts taught in an informal way without actually introducing geometry as subject. From class 6th, geometry is introduced and taught in a formal way, class 7th is first class after transition from primary to upper primary. To see the development and changes in the geometrical concepts class 7th was selected. The study was conducted in two phases. In Phase 1, In order to invoke what 'Triangle' and 'Circle' bring to mind of students, four activities (Drawing, Brainstorming, Defining, Identification) were prepared (based on the study done by Vighi in 2005). To explore the other factors which may contribute to the developmental changes in the mathematical concept, classroom observations, teachers' interview were conducted and information about the students' backgrounds were gathered. Selected students were of low economical status. Parents of most of the students were uneducated and most of the students were sent for mathematics tuition in their nearby place which were not of very good standard as tuition fee of these centers were very low (around Rs.150 to Rs. 200 for all subjects).

In the second phase, in-depth interviews of 20 selected students were conducted. These students were selected on the basis of their responses given in the activity sheets.

Description of tools

Activity Sheets for Students

Activity One: Drawing

Students were asked to draw 4 different triangles and circles, using a sheet of blank paper and they were free to draw either using geometrical tools or free hand.

Following tasks were carried out:

- Draw a triangle/ Circle
- Draw a different triangle/Circle
- Draw another different triangle/ Circle
- Draw another triangle different from all three.
- The children were also asked to explain the choices they made in their drawing, and to explain the differences.

Activity Two: Brainstorming

In order to further investigate, brainstorming activity was carried out. Each child was asked to think of a word Triangle/Circle and draws everything that the word suggested and they see around.

Activity Three: Defining

Children were asked to give written explanation of what they mean by 'Triangle' and 'Circle' in their own words.

Activity Four: Identification

Children were asked to look at different shapes and their features, identify their main characteristics and decide whether those are triangle or non triangle corresponded to the explicit or implicit definition of a triangle.

Class Observations

Selected sections of classes 5 and 7 of three selected schools were observed for 2-2 days to get a feel of the general pedagogical practices in mathematics followed in the class and to get familiar with the students. However, these class observations were not specific to geometry teaching but to observe the class environment, interaction between student and teacher, students' participation in the class and resources used by teacher in the class to teach mathematics.

Semi-Structured Interview for Teachers

Teachers were asked about general information viz. name, classes taught, academic qualification, professional qualification, experience and school name.

In the 2nd part of the interview, teachers were asked about the following dimensions:

- The resources available in school for mathematics teaching
- Responses from parents of school children
- Books referred by teachers to teach mathematics
- Book followed in class to teach mathematics
- How do they introduce a chapter generally

Questionnaire for Students

There were two sections in the questionnaire along with the general information section. In general information section the student had to fill: name, class, age and school name. Sections were divided dimension wise and comprised of both closed and open ended questions.

Phase 2 comprised of in-depth interview of students on the basis of the analysis of responses given in the activity sheets of phase 1. 10 students of grade V and 10 students of grade VII were selected for the in-depth interview on the basis of variations in the answers.

Results

Some of the major findings based upon four activities given to the students related to the development of the concept triangle are as following:

- Most of the students gave explanation about the difference that was based upon perceptual aspect of geometrical orientation i.e. vertex with changing directions. For example, in the in-depth interview:

Question: *Aap ko ye triangles alag kyo lag rahe hai?*

Answer: *Ek ki chonch upper hai, dusre ki neeche hai*

- Another significant aspect is that the need for diversification led a number of children to change the sides of the triangle. They obtained what they referred to as a 'moved', 'inclined' triangle', or a triangle with curved sides or even, more surprising, a triangle where outer sides are designed. Thus, many children do not have the idea of side or rather they have, a very specific idea, certainly not Euclidean.
- The most significant result, however, was that the triangle is essentially the same, equilateral and with one side horizontal. It is "the" triangle; some children call it the "normal/simple triangle". Students have a perception about triangle which should have a horizontal base and a point. The plausible explanation is that this type of triangle is usually drawn by their teacher or most of the time it is in their textbooks.
- Looking at the changes from class V to class VII there is not much difference found in the drawing of triangles as most of the students drew acute angled triangle which is considered as prototype (Hershkowitz, & Bruckheimer & Vinner , 1987) of the triangle except very few students of class VII who drew right angled triangle as different one.
- Looking at the lexical aspects i.e. language used by students, it can be seen that more students of class VII used more propositions instead of using natural speech for the explanation given to make different choices e.g. students of class VII could differentiate in terms of types according to sides i.e. scalene, isosceles and equilateral but they could not use the same criteria for drawing activity so there is a plausible explanation that students tried to write the rote memorized answers which were taught in the class.
- Only 2% of class VII students could define triangle, but most of the students gave partially correct definition. Their explanation includes the following attributes :
    - It is a shape with three peaks(three angles).
    - Triangle has three sides.
    - It has three corners.
    - Another explanation is horizontal line and a peak.
    - Class VII students also tried to explain triangle by giving its types viz. scalene, equilateral and isosceles triangle and explaining them.
    - One line horizontal and other two lines are inclined.
- Another aspect found was that figure D was identified as a triangle by all the subjects questioned. This is, certainly, the prototype of the triangle concept (what may be called specifically an 'equilateral' triangle).
- Most of the class V students were not able to handle the compass to draw circle.
- Another significant aspect that emerges which is consistent with what Duval (1998) explains about three cognitive processes i.e. student could not draw equilateral triangle though he knows what it is.

Other interesting findings based upon the teachers' interview and other resources are:

- Primary school teachers are mother teachers and teach all the subjects so lack of expertise.
- There are no mathematics lab facilities available for the students of classes V and VII.
- No interactions between primary school teacher and upper primary school teacher about the pedagogical practices.
- Students belong to poor families and parents are not educated still they go to mathematics tuition.
- There is possibility that these tuitions are not of good standard, so students instead of understanding the concept tried to give memorized answers.

Major findings related to the Development of the concept 'Circle' are as following:

- Students have a vague idea about the word 'round', round for them means without any cut and sharp point or a round figure is one that does not have edges.
- Another important aspect is that Circle is considered as round figure with no shape. It means for them shape means a sharp pointed with proper edges.
- Students could not think of the formal definition of circle, though it was not taught at that level. So it seems important in case of circle that formal definition of circle needs instruction i.e. mediation is important and it does not depend upon age.
- Class V students could not handle the compass to draw circle they either drew it free hand or with using any round shape. However class VII students used compass to draw circle.
- Another significant result is that first circle drawn by most of the students was perfectly round but 2nd and 3rd circle was made either in oval shape or with different outer designs in order to make them different. First circle like triangle was considered normal circle. So the word 'different' made them think to distort the actual shape of circle.
- Looking at the changes from class V to class VII there is not much difference found in the drawing of circles however while explaining few students of class VII used the criteria of difference in radius. Not even a single child could give a formal definition of circle so it shows importance of instruction in spite of age as factor for the development of the concept.

- As far as lexical aspect is concerned, students used explanation in natural speech e.g. circles are different because a) they are different in size b) they are different shapes like moon, sun c) they are made using different instruments.
- Students are not able to define circle without being taught however they were able to identify it correctly, plausible explanation is that as circle is very close to real life objects so figural aspect is stronger than the conceptual one.

Discussion and Implications for Teaching and Learning Geometry

As far as the development of the understanding of these concepts is concerned, researcher found a huge gap between the personal and geometrical meaning of the concepts among students of class V and VII. Though there is an improvement in the answers of class VII students than class V, analysis of the data shows that instruction provided in the class may be responsible for the quality of answers with regard to the control exerted by the conceptual constraints on the figural interpretation of geometrical entities as the attributes explained by the most of the students while defining 'Triangle' were not being used while identifying those figures. Analysis of the data also shows that class V and VII students still belong to Geometrical Paradigm 1 (Houdement and Kuzniak, 2003), as responses of most of the students were based more on intuition and experience than on properties and definitions. Here to note that as per the three paradigms of geometry, students should be able to use geometrical instruments in paradigm 1, but most of the class V students were not able to handle the compass. Another significant aspect that emerges which is consistent with what Duval (1998) explains about three cognitive processes i.e. student could not draw equilateral triangle though he knows what it is. It means development of reasoning or visualization does not assure the development of construction.

As far as **implications** of the findings of the teaching of geometry are concerned, Teaching and learning geometry depends upon the epistemological nature and the cognitive processes involved in understanding geometry. While discussing its epistemological nature, two aspects have been seen. On one hand, it is the study of logical relations, deductive reasoning and on the other hand it refers to spatial concepts, intuitive understanding.

The goal of geometry learning should be the realization of geometry as a deductive structure, with geometry as the science of our environment as a necessary per-requisite. Findings of the study helps to decide the extent of depth of mathematical knowledge and reasoning teachers need for teaching. Based upon these aspects and the cognitive process involved in the development of geometrical thinking, it can be inferred that primary teachers should be at least, in the second paradigm (geometry II), and secondary school teachers should be in third paradigm (geometry III) to help making the transition easier from one paradigm to the other (Dorier, Gutierrez, Strasser, 2003)

Van Hiele levels also provide a good reference to answer the question on reasoning: teachers should reason at least one level higher than students although they have to be aware that they should interact with their pupil on their level. Then primary school teachers should be atleast in the third level and secondary school teachers should be at least in the forth level. Therefore, teacher training courses should allow prospective teachers to learn the relationship among abstract geometrical concepts or properties and their concrete representations, and to discover geometric models in children's ordinary life environment.

- Teachers should focus on different kinds of triangles besides just introducing prototype of triangle. The process of construction and reasoning should be developed separately as they are independent of each other keeping in mind the synergy of these processes is necessary for proficiency in geometry. (Duval, 1998).
- There were some triangles (N,S,T,U) which most of the students could not identify as triangles, some of the reasons they gave are: N because its one side is standing straight and they consider a triangle with horizontal side and two inclined sides, S because it is touching horizontal; T because it is very thin, U earlier was not accepted by many students as they were thinking that it does not have horizontal base but later in the in-depth interview almost all children accepted it. Teachers should focus on these types of triangles besides just introducing prototype of triangle.
- As the analysis shows that students marked those shapes as triangle which are not triangle in the Euclidean space but could be triangle in other space like in non Euclidean space e.g. B and C are triangles in non Euclidean space; teachers needs have knowledge about the different type's geometry.
- Transition from geometry I to geometry II is important and crucial Therefore, teacher training courses should allow prospective teachers to learn the relationship among abstract geometrical concepts or properties and their concrete representations, and to discover geometric models in children's ordinary life environment.
- As Euclidean geometry is most prevalent at school level so it is essential for the teachers to know its triumphs and limitations. It will help to realize the objectives of teaching. At lower level we try to relate students with the physical reality, teachers must know that in Euclid geometry we use ideal object as approximation while applying to our space. It is just a matter of convenience. As recent developments in psychological view of geometry have also shown the importance of spatial reasoning, Euclidean geometry has not remained just a fixed body of axioms and postulates and deductive

reasoning. Since then the content of this geometry has been revised by many modern mathematician.

- To understand the children's understanding of geometric thinking - Other theoretical frameworks (alternative routes by Battista, 2007 and Houdements and Kuzniak' theoretical framework, 2003) which are built up to elaborate or revise Van-Heile levels help to provide more elaborate understanding of children's understanding of geometry i.e. how does it develop in small incremental steps and progress through transition phases.
- To decide the appropriate tools for the learning of geometry Van Hiele levels give importance to teaching and using instruments to develop geometrical reasoning as at level I, II, III there is need of physical objects, instruments and drawing to help students solve task or understand geometrical structure or organize their reasoning. Technology is also being used for the learning of geometry as it focus on the aspects of visualization and spatial reasoning which are important aspects of geometrical thinking. Laborde; Kynigos; Hollebrands; Strasser (2006) found dialectical link between the development of theories and research on the use of technology in geometry.
- Major implications of Duval's theoretical framework:
  - The three kinds of processes i.e. visualization, construction and reasoning must be developed separately. This framework says that they are independent of each other
  - As there are various kinds of reasoning and various ways to see a figure so it is important for teachers to use differentiated task for the students to develop various kinds of reasoning and visualization.
  - Those tasks also be involved by which there processes can be used in co-ordination as "their synergy is cognitively necessary for proficiency in geometry" (Duval, 1998).

Some of the plausible explanations related to the pedagogical practices and lack of resources related to mathematics teaching might be the factors responsible for lack of understanding of Geometry. Observation of class V and VII shows that Primary school teachers are mother teachers and teach all the subjects so there is a lack of expertise. There are no mathematics lab facilities available for the students of V and VII. Another aspect is that most of the students belong to poor families and parents are not educated still they go to mathematics tuition; there is possibility that these tuitions are not of good standard, so students instead of understanding the concept tried to give memorized answers. But more investigation is required pertaining to the same.

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Insights into Student Errors in Ordering of Fractions, Equal Sign, Interpretation and Identification of Shapes

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Abstract: In this paper we shed light on the student errors or alternate conceptions students have in ordering of fractions, equal sign interpretation and ordering of shapes. We show the pattern in the extent of the wrong notion across age groups. We use the response data from students of classes 4, 6 and 8 mainly for this. We also shed light on the presence of the wrong notion and its extent among students of different ability levels. We use the data from large scale studies administered across different student groups of English medium private schools of India to provide such granular information. The chapter, Insights into Students' Errors Based on Data from Large Scale Assessments, to be published in the First Asian Sourcebook on Maths education: Korea, Singapore, China, Japan, India (Shah, 2012) throws light on a few student errors and patterns across age-groups. The paper extends the work done by these authors and sheds light on few more common student errors in the said areas. For the scope of this paper, the data available is mostly in the form of responses on multiple-choice questions answered by students of different class levels (3 – 9).

Introduction

Often students already have pre-instructional knowledge of a topic when a teacher is providing instruction on concepts. A teacher should ideally build upon these ideas. Students' pre-instructional knowledge, however, can be erroneous or misinformed. These erroneous understandings are termed as alternative conceptions or misconceptions. Some of the misconceptions (alternative conceptions) may hamper building of other math concepts and give rise to new misconceptions. For example, knowing only operational meaning of equal sign will come in the way of learning to solve linear equations later as the relational meaning of equal sign helps to manipulate linear equations and solve them (Knuth, 2006). There are some common alternative conceptions that most students typically exhibit. A few of them are shared in the paper. It would be useful for a teacher to be aware of common student errors or alternative conceptions (misconceptions). It would also be useful to know which alternative conceptions go off and which continue to remain across age groups. Research in mathematics education could aim at developing instructional strategies which are effective in bringing conceptual change and help students leave their alternative conceptions behind and learn correct concepts or theories. An in-depth understanding of errors and the underlying student ideas, their prevalence across grades, and the likelihood of their occurrence in different groups of students is a critical part of a teacher's pedagogical content knowledge (Shulman, 1987).

Assessments which are diagnostic and can reveal students' thinking are being developed. The individualized feedback provided by them about student errors or alternative concepts can help teachers personalise learning by planning lessons to meet the individual needs of students in their classes. In India, the research on student errors so far has been largely based on classroom observations and interviews with individual or a group of students. With the improvement in data collection technologies in the last two decades, it has been possible to find students' responses for a large number of questions across the grades. Benchmarking studies on student learning standards and diagnostic tests in each subject are also being carried out globally (Loveless, 2007). Student errors and incorrect notions are captured in test items with highly attractive multiple choice distracters (Sadler, 1998). The distracters are developed to capture the error-pattern that students have which are already identified through previous research or which they are likely to have. The data from such large scale studies administered across demographies and different student groups serve as a repository for finding common patterns in errors that students make, and provide granular information of student knowledge. The book, Compendium of Errors in Middle School Mathematics by Pradhan and Mavalankar provides a limited compilation of Indian students' errors (Pradhan, 1994). The chapter, Insights into Students' Errors Based on Data from Large Scale Assessments, to be published in the First Asian Sourcebook on Maths education: Korea, Singapore, China, Japan, India (Shah, 2012) throws a light on a few student errors and patterns across age-groups. The paper extends the work done by these authors and sheds light on few more common student errors.

Data collection

For the scope of this paper, the data available is mostly in the form of responses on multiple-choice questions answered by students of different grade levels (4 – 9). For the purpose of this chapter, the student response data from the following sources have been used:

Data from Wipro Quality Education Study[1] (2011): The items are taken from a common test of 22 questions which was administered to class 4, class 6 and class 8 students.

ASSET® - developed by Educational Initiatives Pvt. Ltd., is a diagnostic test taken by about 450,000 students every year across different classes of private English medium schools in India in 5 major subjects including Maths. All the students of different ability groups of participating schools take the test. Students from Indian schools staying abroad also take this test. The data is taken from the tests taken by students in the last 4 years.

The analysis of the data from these sources can confirm or contradict the findings from small-scale studies leading to more fine-grained insights about student errors and the pedagogy of specific topics, especially in the Indian context.

Student errors in Ordering and Comparison of Fractions

The understanding that fractions are also numbers and hence the skill to order them is an important part of concept of fractions. The response data on questions in this section will shed light on students' understanding of fractions as a number.

The following is a question asked to class 4, class 6 and class 8 students of English medium private schools of 5 metros as a part of Quality Education Study.

Item 2.1

$\frac{5}{5}$ □ $\frac{4}{4}$

What should come in the empty box to make the number sentence true?

A. =
B. <
C. >
D. (We cannot say for sure.)

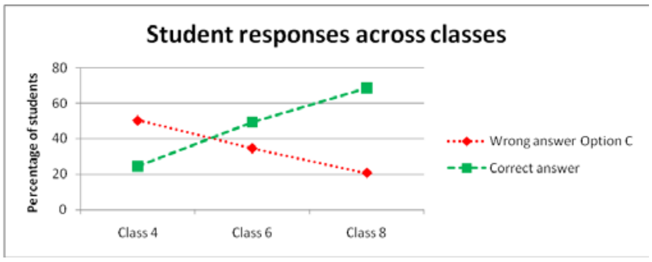
Student response data Table 2.1

Classes	No. of students	Option A (in %)	Option B (in %)	Option C (in %)	Option D (in %)
Class 4	1556	24.5	9	50.4	9.5
Class 6	1816	49.4	7	34.6	6.2
Class 8	1559	68.8	4.4	20.6	5.3

(Percentage of students answering correctly is shown in bold. Answer option C is the most common wrong answer option and the corresponding column is shown shaded in grey. We use the same convention throughout the paper.)

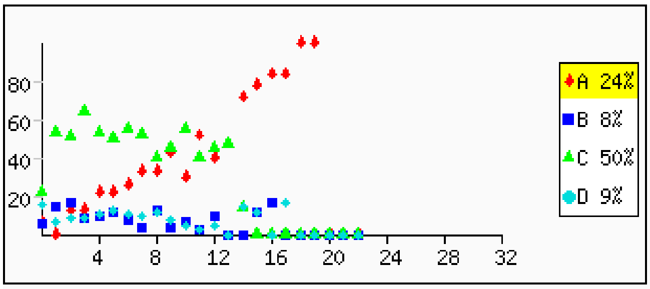
As per the National Curriculum Framework (NCF) – 2005, students of class 4 are expected to know the fact that a fraction of the form $\frac{a}{a} = 1$. As seen from the figure 2.1 the percentage of students answering correctly increases linearly as one moves from class 4 to class 8. The percentage of students answering as $\frac{5}{5} > \frac{4}{4}$ is also decreasing linearly as one moves from class 4 to class 8. However, about $\frac{1}{5}$ of class 8 students continue to have this wrong notion.

Figure 2.1



Item Response Curves:

Figure 2.11 Class 4



[1] This is a research study undertaken by Educational Initiatives (EI) and Wipro Ltd. in the 5 metro cities – Mumbai, Kolkata, Chennai, Delhi and Bangalore and a few schools with different learning environment. http://www.ei-india.com/wp-content/uploads/Main_Report.pdf

Figure 2.12 Class 6

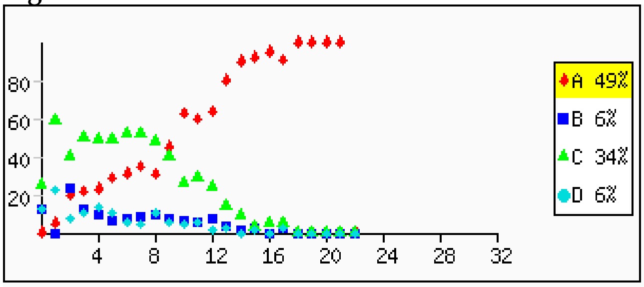
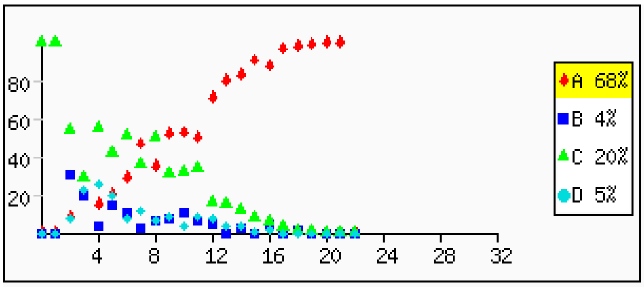


Figure 2.12 Class 6



The item response curves indicate that students of average and below average mathematical ability across classes have the wrong notion. Few brighter students also have this wrong notion. These students seem to think that higher the numerator and/or denominator greater the fraction as they extend whole number ordering falsely to fractions. Most of these students may have thought that $\frac{5}{5} = 5$ and $\frac{4}{4} = 4$. As $5 > 4$, $\frac{5}{5} > \frac{4}{4}$. A few students would wrongly apply ordering of whole numbers to find both the numerator and the denominator greater in $\frac{5}{5}$ than in $\frac{4}{4}$ to answer as $\frac{5}{5} > \frac{4}{4}$. The following responses of few students of class 6 from an English medium private school in Gujarat, whom we interviewed on this question, support this fact.

Student A: $\frac{5}{5}$ is 5 wholes and $\frac{4}{4}$ is 4 wholes. $5 > 4$ and hence $\frac{5}{5} > \frac{4}{4}$.

Student B: $\frac{5}{5} > \frac{4}{4}$ as both the numerator and denominator are greater that the numbers in $\frac{4}{4}$.

Student C: $\frac{5}{5}$ is one whole which is 5 and $\frac{4}{4}$ is 1 whole which is 4!

To see what proportion of students see a fraction as two different numbers, we share the response data of the following ASSET question asked to class 6 students. The response data show large number of students think that a fraction is a number that lies between the numerator and the denominator. Less than 15% of the entire class 6 test takers knew that $\frac{110}{570}$ is less than 1.

Item 2.2 Class 6

$\frac{110}{570}$ lies between _______________.

A. 0 and 1
B. 1 and 2
C. 11 and 57
D. 110 and 570

Student response data Table 2.2

N = 15543	Option A	Option B	Option C	Option D
% students	14.7%	8.10%	42.60%	32.90%

The response data on the following question gives an idea about the state of the assimilation of the concept of fractions as a number at class 7 level. To discourage students to take the least common multiple (LCM) of numbers and then order, the numbers in the fractions are given large. Students are expected to reason that an improper fraction is greater than 1, a proper fraction is less than 1 and a fraction of the form $\frac{a}{a}$ equals 1. They are expected to apply the understanding to identify the greatest among the fractions, $\frac{7989}{7988}$, $\frac{7988}{7989}$, $\frac{7989}{7989}$ and $\frac{7991}{7993}$.

Item 2.3 Class 7

Which of these numbers is the greatest?

A. $\frac{7989}{7988}$

B. $\frac{7988}{7989}$

C. $\frac{7989}{7989}$

D. $\frac{7991}{7993}$

Student response data Table 2.3

N = 14574	Option A	Option B	Option C	Option D
% students	**26.6%**	8.4%	21.6%	42.0%

Only about 27% of the students of class 7 could identify that $\frac{7989}{7988}$ (improper fraction) is greater than 1. About 22% of the students think that $\frac{7989}{7989}$ is greater than $\frac{7989}{7988}$ and is the greatest among the given numbers. Most of these students may also be having the notion that $\frac{7989}{7989} = 7989$ and hence say $\frac{7989}{7989}$ is the greatest among the given fractions. The student response data also shows that 42% of the class 7 students of English medium private schools feel the fraction with higher numerator and/or denominator is greater than the fraction with smaller numerator and/or denominator. This happens when students inappropriately extend their understanding of ordering of natural numbers to order fractions. The research done by Harnett and Gelman (Harnett, 1998) following earlier research (Behr M. J., 1992) shows that one of the reasons why the notion of fractions is systematically misrepresented is because it is not consistent with the counting principles that apply to natural numbers. The study done by Stafylidou et al. (Stafylidou, 2004) reveals how such misconceptions on ordering of fractions get generated among students when they attempt to relate the information they receive about fractions with their prior knowledge and their dynamic aspect. Some researchers (Behr M. J.,

1992) (Lamon, 1999) (Stafylidou, 2004) stress the need for more practice on partitioning and measurement activities and giving activities that relate the unit to fractions, which they believe will help students develop the concept of the unit and thus help order and compare fractions.

In the Indian context, emphasis laid mainly on procedural understanding of ordering of fractions does not help to develop the sense of fractions as a number.

Understanding of Equal Sign

Over 20 years of research in developmental psychology and mathematics education has indicated that many elementary school students (ages 7 to 11) have an inadequate understanding of the equal sign (Baroody, 1983) (Baroody, 1983) (Behr M. E., 1980) (Carpenter, 2003) (Rittle-Johnson, 1999) (Kieran, 1981) (McNeil, 2005). Instead of interpreting it as a relational symbol of mathematical equivalence, most students interpret the equal sign as an operational symbol meaning “find the total” or “put the answer.” Students not only provide operational interpretations when asked to define the equal sign, but also rate operational interpretations such as “the total” and “the answer”, as “smarter” than relational interpretations such as “two amounts are the same” (McNeil, 2005).

The response data on the following item from TIMSS 2007[2] asked to Indian students as a part of the Quality Education Study attempts to shed light on meanings of equal sign interpreted by Indian students.

Item 3.1

12 ÷ 3 = ■ ÷ 2

In this number sentence, what number does ■ stand for?

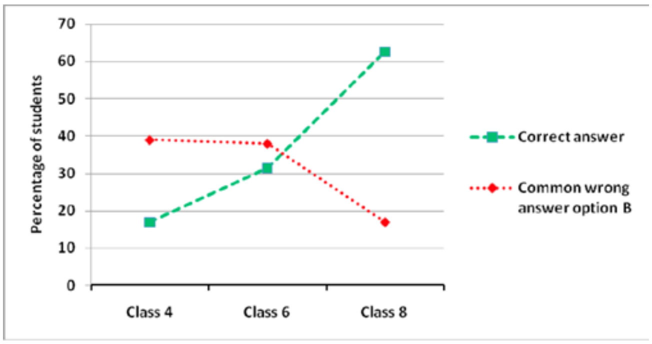
A. 2
B. 4
C. 6
D. 8

The item tests whether children understand that ‘=’ sign means equivalence of expressions on its either side and not a symbol ‘to write answer after’ or ‘an operator which means do something’.

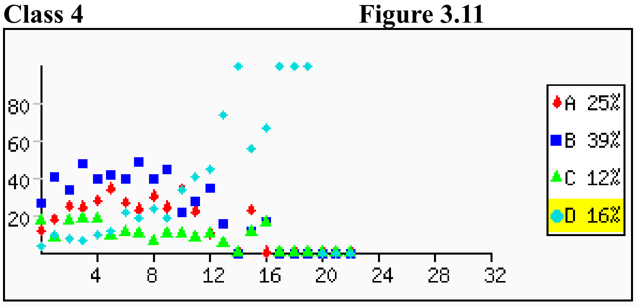
Student response data — Table 3.1

Class	Number of Students (N)	Option A (in %)	Option B (in %)	Option C (in %)	Option D (in %)
4	1556	25.0	39.0	12.5	**16.9**
6	1816	20.4	38.0	8.0	**31.4**
8	1559	14.4	16.9	3.7	**62.6**
TIMSS grade 4	-	9.7	46.7	12.5	**26.7**

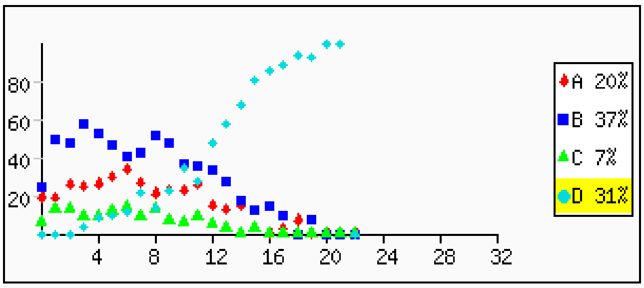
Figure 3.1



Class 4 **Figure 3.11**

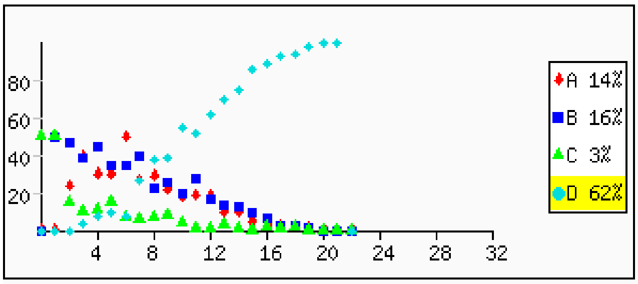


Class 6 **Figure 3.12**



[2] Copyright © 2009 International Association for the Evaluation of Educational Achievement (IEA) Publisher: TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College

Class 8 **Figure 3.13**



Students answering B are clearly interpreting equal sign as the 'answer of' the expression to its left. These students seem to ignore '÷ 2' from RHS. Students answering as A seem to be interpreting the given number sentence the same as 12 ÷ 3 ÷ 2 = □. It can be seen that sizeable number of students have a misconception that equal sign is an operator which means 'answer of'. About 64% of class 4 students have shown to have the misconception. Also, even in class 6, the percentage of students having this misconception doesn't change much (about 58%) although the percentage of students who answered the question correctly almost doubles. It is only in class 8 that the percentage of students having the misconception reduces (about 30%).

The student response data on the following items of ASSET adds to our understanding of the misconception.

Item 3.2 **Class 4**

220 + 380 = _____ + 400
What should come in the blank above to make the number sentence true?

A. 200

B. 380

C. 600

D. 1000

Response data **Table 3.2**

Class	Number of Students	Option A	Option B	Option C	Option D
4	26267	31.5%	9.9%	33.7%	14.3%

About 50% of class 4 students (students answering C or D) exhibit operational meaning of'='sign being an operator. Students answering C have ignored + 400 in R.H.S. Only 31.5% of the students seem to understand relational meaning of = sign and have answered correctly as A.

The student response data on a similar ASSET item shows students in class 5 as well exhibit operational meaning of equal sign. Only 5% more students exhibit relational meaning of equal sign.

Item 3.3 **Class 5**

231 + 50 = _____ + 80

What number should come in the blank above to make the number sentence true?

A. 191

B. 201

C. 281

D. 261

Response data **Table 3.3**

Class	Number of Students	Option A	Option B	Option C	Option D
5	17450	4.2%	36.5%	42.7%	15.2%

About 36% of class 4 students don't consider 10 = 10 and 10 = 5 + 5 as correct. About 27% of students don't consider 10 = 10 to be correct. These students can be said to lack the proper (relational) meaning of equal sign.

Item 3.6 **Class 5**

Balbir, Gunjan and Alya have written 3 number sentences.

Balbir: 815 = 800 + 15
Gunjan: 800 + 15 = 815
Alya: 800 + 10 + 5 = 815

Whose number sentence is correct?

A. only Gunjan's

B. only Alya's

C. both Gunjan's and Alya's

D. All of them are correct?

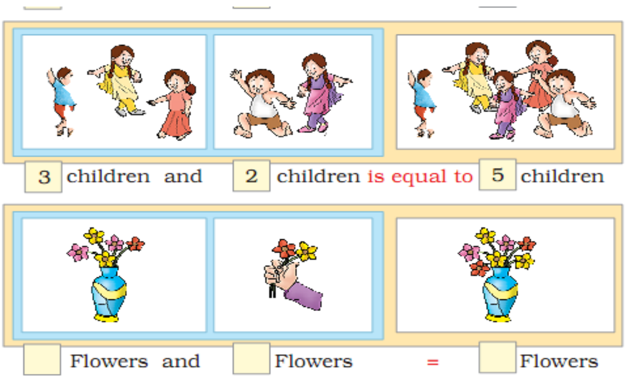
Response data **Table 3.6**

Class	Number of Students	Option A	Option B	Option C	Option D
5	9090	10.6%	16.6%	32.0%	39.5%

Only about 40% of the class 5 students consider all the 3 number sentences to be correct and understand the meaning of equal sign correctly. The rest don't consider Balbir's statement 815 = 800 + 15 to be correct.

In India, National Council for Educational Research and Training (NCERT) textbook introduces 'equal to' sign as a replacement for 'is equal to' in class 1.

Figure 3.6



NCERT Math Magic textbooks of classes 2 to 5 don't give sufficient exposure to number sentences of the form *a = a, a = b # c and a + b # c + d, where # is any of the 4 basic operations*. Lack of adequate exposure to such number sentences may be the main reason why students fail to interpret equal sign as 'balance' and interpret it as 'answer of'.

To counteract this misconception, a balance scale can help students develop the proper conceptual understanding of equality and the equal sign (Reys, 2004). Van de Walle (Walle, 2004, p. 139) suggests that teachers should use the phrase "is the same as" instead of "equals" as students read number sentences.

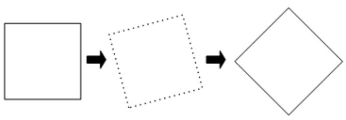
A study of the same misconception in China and U.S. showed that about 98% of class 6 students of China were able to answer correctly as opposed to only 28% of their U.S. counter-parts (Xiaobao Li, 2008). Chinese textbooks typically introduce the equal sign in a context of relationships and interpret the sign as "balance," "sameness," or "equivalence" and only then embeds the sign with operations on numbers.

Understanding of Basic Shape

The item 4.1 that follows checks whether students identify shapes based on their properties or merely match their mental image of shapes mainly in a standard orientation. They are expected to see that an orientation of a shape is an irrelevant attribute of the shape.

Item 4.1

Afzal has made a square on his computer screen. He now turns the shape as shown. What is the change in the shape?



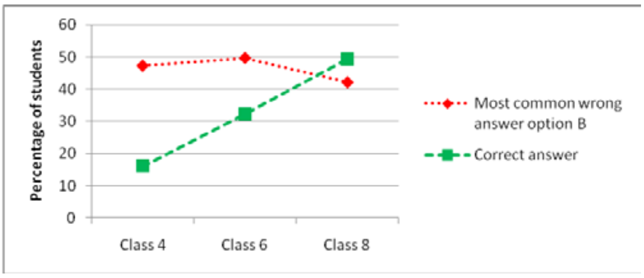
A. The square changes into some other shape and its side lengths also change.
B. The square changes into some other shape but its side lengths don't change.
C. The figure remains a square, but its side lengths change.
D. The figure remains a square, and there is no change in its side lengths.

Student response data **Table 4.1**

Class	Number of students	Option A (in %)	Option B (in %)	Option C (in %)	Option D (in %)
4	1556	15.4	47.3	13.3	**16.1**
6	1816	6.6	49.7	8.1	**32.2**
8	1559	2.4	42.1	4.2	**49.3**

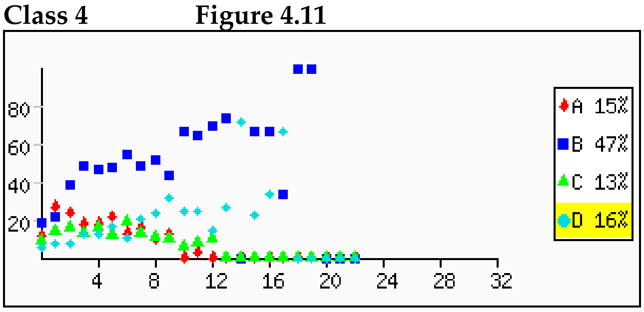
The students choosing option B think that the square shape changes when it is rotated. These students are identifying the shape visually based on the shape in the orientation that they are familiar with than using its properties. The class 6 and 8 students might be thinking that the shape has changed to rhombus and is no longer a square, as they may be familiar to a rhombus is such an orientation.

Figure 4.1

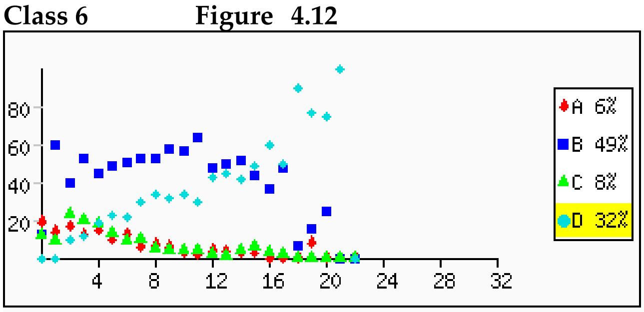


The data across the three classes shows that the percentage of students answering correctly has increased from 16.1% in class 4 to 49.3% in class 8. The most common wrong answer option for all the three classes is option B. Interestingly the percentage of common wrong answer doesn't reduce much as we go from class 4 to class 8. It can be seen from item response curves, given below, the wrong notion persists among students of all ability groups except among very bright students.

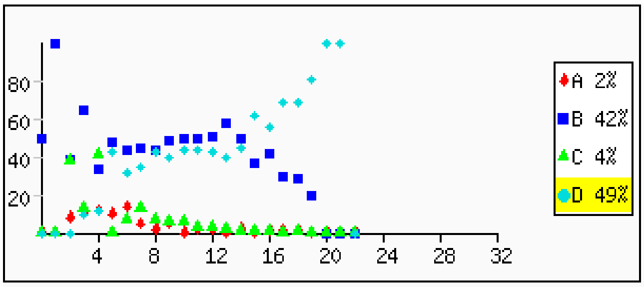
Class 4 **Figure 4.11**



Class 6 **Figure 4.12**



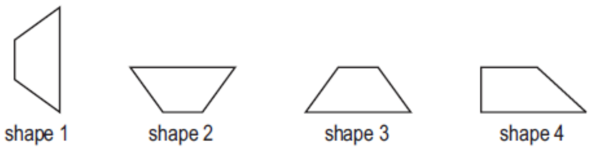
Class 8 **Figure 4.13**



The Van Hiele Levels of reasoning describe the students understanding and reasoning ability in geometry (Crowly, 1987). Identifying shapes in different orientations is a characteristic of the second Van Hiele level (level 1). Students of class 8 are expected to prove simple results which is a characteristic of the fourth Van Hiele level (level 3). But we can see that 50% of the class 8 students are at first level (level 0) when it comes to identifying shapes based on their properties. The response data of the following item from ASSET also suggest that students are not at the expected Van Hiele levels.

Item 4.2

Which of these shapes is/are trapezium (s)?



A. shape 3 only

B. shapes 1 and 3 only

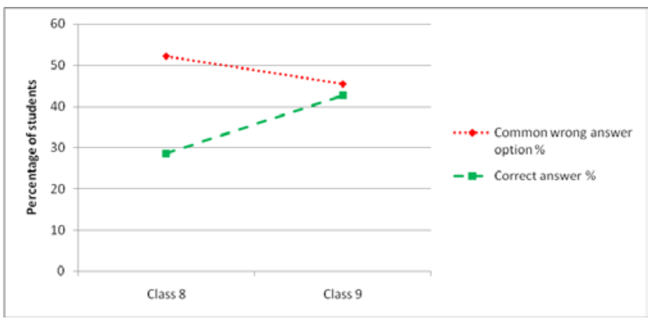
C. shapes 1, 2 and 3 only

D. shapes 1, 2, 3 and 4

Response data **Table 4.2**

Classes	No. of students	Option A (in %)	Option B (in %)	Option C (in %)	Option D (in %)
8	21940	10.7	7.4	52.2	**28.6**
9	17329	6.4	4.3	45.5	**42.7**

Figure 4.2



The student response data suggests that many students are identifying shapes based on visual appeal than using properties to identify them. Students who answered A are unable to see that an orientation is an irrelevant attribute of the shape. These students are thinking at first Van Hiele level (level 0). Almost 50% of the students answered C in both the classes 8 and 9. These students could not recognise shape 4 as a trapezium. They may not have realised that two opposite sides being parallel is a sufficient condition for a quadrilateral to be a trapezium. This shows that these students are still at second Van Hiele level (level 1). They seem to have a notion that a trapezium cannot have any right angle. The student response data of the following item shows the same.

Item 4.3 **Class 9**

PQRS is a trapezium with one of its angles a right angle. Which of these is true about the trapezium?

A. It has one more right angle

B. None of its angles can be obtuse angles.

C. A pair of its opposite sides are equal in length.

D. (Such a trapezium with one of its angles a right angle is not possible.)

Response data **Table 4.3**

No. of students	Option A	Option B	Option C	Option D
7734	**38.9%**	14.5%	20.1%	25.2%

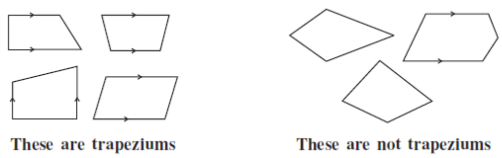
About 25% students thought that it is not possible to have a trapezium with one of its angles a right angle.

It is important that students are exposed to shapes in different orientations, so that they realise that orientation of a shape is an irrelevant attribute. The shape remains the same irrespective of its orientation. A triangle remains a triangle irrespective of its orientation in a plane. It can be noted that NCERT class 8 textbook (chapter 3) does give an example of a trapezium with a right angle as shown below.

Figure 4.3

3.4.1 Trapezium

Trapezium is a quadrilateral with a pair of parallel sides.



Study the above figures and discuss with your friends why some of them are trapeziums while some are not. (**Note:** *The arrow marks indicate parallel lines*).

Future Research Implications

Further studies can be taken up to know what pedagogical approach works in reducing the extent of the alternate conceptions discussed in this paper. The insights can be disseminated widely among teachers and can be made a part of their professional development. The impact of sharing the insight on students' alternate conceptions with teachers can be measured by taking a treatment group of teachers with whom such information is shared and taking a control group. The change in student misconception pattern can be measured to do this. Comparative analysis of student errors and the pattern among age groups can be done for other important topics of Maths than those discussed in the paper. Impact of the misconception of equal sign on solving of a linear equation by the students who have the misconception could be measured.

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Students' Solving Processes of Linear Equations at Elementary Stage

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Students' solving processes of linear equations at elementary stage

Algebra is a branch of mathematics concerning the study of structure, relation and quantity. The concepts, principles, and methods of algebra constitute powerful intellectual tools for representing quantitative information and then reasoning about that information. Elementary Algebra is the most basic form of algebra. It is taught to students who are presumed to have no knowledge of mathematics beyond the basic principles of arithmetic. The central concepts of algebra include variables, functions, relations, equations, and inequalities, and graphs. An equation can be considered as an open sentence of which the domain of validity has to be determined (with the point of view of "variable"). It is also considered as a condition defining one or several numbers (with the point of view of "unknown"), and in this case, there are only right equalities. An equation can be considered two expressions set equal to each other. And solution of the equation means finding a value, such that, when you replace the variable with it, it makes the equation true i.e. the left side comes out equal to the right side. Generally the students easily learn the way to get a solution of the equation but they do not understand the process itself and why they are doing in that process.

Kieran (1989) pointed out that the traditional emphasis in the curriculum on finding the answer allows learners to get by with informal, intuitive procedures. Kieran (1992) contended that students do not view the equal sign as a symbol of equivalence but rather as an announcement of the result or answer of an arithmetic operation. The notion among beginning algebra students that the equal sign is a "do something signal" (Behr, Erlwanger & Nichols, 1976) rather than a symbol of the equivalence between left and right sides of an equation is indicated by their initial reluctance to accept statements such as 4 + 3 = 6 + 1 or 3 = 3. They think that the right side should indicate the answer, that is, 4 + 3 = 7. Children meeting algebra for the first time often have great problems in understanding the meaning of the notation. Booth (1984, 1988) found that the students used letters as labels or as specific values, rather than as variables. One of the findings of Kuchemann (1981) revealed that few students were able to interpret letters as variables and a very small percentage of 13-15 year old pupils were able to consider the letter as a generalized number.

A few researches are concerned to the strategies adopted by the students to solve the linear equations and their relationship with the child's conceptual development. Kieran (1992) describes some strategies such as use of known basic facts, counting techniques, guess and check, cover up and working backward. Sfard (1991) states that children see equations as the description of an arithmetic process with guess and check as a natural way of finding x. Solving the equations by working backwards also shows that students have the view of equations as processes. Sfard and Linchveski (1994) admit that viewing equation as an object is essential in order to solve it. Herscovics & Linchveski (1994) showed that many children revert to the strategy of guess and check to solve equations e.g. 2x + o the students who solve it through guess and check. But the simple additive part-whole or multiplicative part-whole thinking is not enough to solve the equations of the kind 2x+3=11, rather it require an understanding of numbers beyond it. Linsell (2009) mentioned that there is a hierarchy of sophistication of strategies and found that many students were unable to solve the equatiy be associated with some solving process.

In India, a good number of studies have been done to find out the achievement in mathematics education as well as the causes responsible for low achievement of students in mathematics education. But hardly a few studies have been done to investigate the understanding of mathematics among students.

Methodology

This study was undertaken to investigate how seventh grade students solve linear equations with one variable and to infer their thinking underlying their solution processes. Six categories of linear equations were used, namely equations involving addition and subtraction, equations involving multiplication and division, equations involving the grouping of the numerical terms, equations involving both additive and multiplicative operations, equations involving double occurrence of unknown on the same side, equations involving double occurrence of unknown on the both sides. Forty (40) seventh grade pupils were involved as the participants of this study. A set of problems (24 linear equations adopted from Herscovics & Linchevski, 1994) was administered on the students to investigate the strategies adopted by them to solve equations. The investigator conducted interview on the students individually for in-depth analysis of the same. Data were collected via the participants' written solutions, think-aloud verbal protocols, retrospection through task-based interview.

The data is analyses on the basis of six categories such as equations and a coding system is used classify the solution strategies of the students. The categories of the equations are such as:

1. Equations involving addition and subtraction
2. Equations involving multiplication and division
3. Equations involving the grouping of the numerical terms
4. Equations involving both additive and multiplicative operations
5. Equations involving double occurrence of unknown on the same side
6. Equations involving double occurrence of unknown on the both sides

The solution strategies adopted by the students are such as:

1. Complementary Subtrahend or factor (C)

For example, in solving an equation like 37-n = 18, generally student adopt a procedure that uses difference as a subtrahend in the complementary form 37- 19 = 18.

2. Division by 2 (Div. 2)

In solving equation like n + n =76, student adopt a procedure called division by 2 such as

$$n + n = 76$$

$$2n = 76$$

$$\frac{2n}{2} = \frac{76}{2}$$

$$n = 38$$

3. Grouping of Numerical Terms(G)

$$14 + n + 17 = 50$$

$$n + 14 + 17 = 50$$

$$n + 31 = 50$$

Student shifts the second number (17) next to the first number(14) to add them.

4. Grouping of Unknown Terms(GU)

$$n + 5 + n = 55$$

$$n + n + 5 = 55$$

$$2n + 5 = 55$$

Student shifts the second 'n' next to the first 'n' to combine them.

5. Inverse Operation(I)

$$n + 5 = 55$$

$$n + 5 - 5 = 55 - 5$$

$$n = 50$$

6. Random Substitution(RS)

$$3n + 4n = 35$$

To solve it, some students try to substitute the value of n randomly such as

$$3 \times 1 + 4 \times 1 = 3 + 4 = 7$$

$$3 \times 4 + 4 \times 4 = 12 + 16 = 28$$

$$3 \times 5 + 4 \times 5 = 15 + 20 = 35$$

7. Systematic Substitution(SS)

$$3n + 4n = 35$$

To solve it, some students try to substitute the value of n systematically such as

$$3 \times 1 + 4 \times 1 = 3 + 4 = 7$$

$$3 \times 2 + 4 \times 2 = 6 + 8 = 14$$

$$3 \times 3 + 4 \times 3 = 9 + 12 = 21$$

$$3 \times 4 + 4 \times 4 = 12 + 16 = 28$$

$$3 \times 5 + 4 \times 5 = 15 + 20 = 35$$

8. Inverse Addition Algorithm(IAA)

In solving 1269 = 693 + n, students may write it vertically and work out the missing added digit by digit such as

$$\begin{array}{r} 693 \\ + \text{------} \\ \hline 1269 \end{array}$$

9. Inverse Subtraction Algorithm(ISA)

In solving 269 = 693 - n, students may write it vertically and work out the missing subtracted digit by digit such as

$$\begin{array}{r} 693 \\ - \text{------} \\ \hline 269 \end{array}$$

10. Substitutes and Succeeds on First Trial(SFT)

In solving 9n - 4n = 35, if students substitute the value of n as 7 in the first trial, it will be called SFT procedure.

Results and Discussion

Table 1: Percentage of students adopting strategies in solving equations involving addition and subtraction

Eq. No.	Equation	C	ISA	SF	RS	IAA
1	14 + n = 43	73	12.5	7.5	2.5	0
2	35 = n + 16	70	12.5	7.5	0	0
3	n – 13 = 24	70	2.5	7.5	0	10
4	37 – n =18	68	2.5	5	5	5
5	17 = n – 15	65	0	5	5	7.5
6	23 = 37 – n	68	7.5	5	2.5	5

C=Complementary Subtrahend, ISA=Inverse Subtraction Algorithm, IAA=Inverse Addition Algorithm, SF=Substitution on First Trial, RS=Random Substitution

Most of the students (91%) solved the equations through complementary subtrahend procedure. Of those students who had adopted complementary subtrahend strategy (C) to solve the equations involving addition, almost all of them successfully solved the equations. While the students who adopted Inverse Subtraction algorithm (ISA) to solve the equations involving addition, half of them could not solve the equations successfully. Of the students who adopted Substitution on First Trial (SF) to solve the equations involving addition, all of them successfully solved the equations involving addition. Of those students who had adopted complementary subtrahend strategy (C) to solve the equations involving subtraction, the success rate of the equations involving subtraction was less than the success rate of equations involving addition. But the students who adopted the Inverse Addition Algorithm (IAA) or Substitution on First Trial (SF) strategy have a good success rate. It seems that the students have a procedural thinking such as they have to subtract the number by shifting from one side of the equal sign to the other and unknown should be always on the left side of the equal sign. That is why the students faced the difficulties in solving the equations having unknown either on right side or as a subtrahend.

Table 2: Percentage of students adopting strategies in solving the equations having multiplication and division.

S.No.	Equation	C	SF
7	16n = 64	63	4.5
8	2088 =174n	60	5.0
9	n ÷ 6 = 13	68	2.5
10	84 ÷ n = 4	68	7.0
11	15 = n ÷ 7	58	7.0

A large number of the students (63%) have solved the linear equations having multiplication and division through complementary factor procedure and some of the students used Substitution on first trial (SF). Majority of the students used complementary factor strategy to solve equations involving one operation multiplication or division. Of the students who adopted the complementary factor strategy (C) to solve the equations involving multiplication, all of them successfully solved the equations. While the students who adopted the complementary factor strategy (C) to solve the equations involving division, the success rate of the students was low in comparison the previous one. It indicates that the students have a procedural thinking such as they have to multiply or divide the number by shifting from one side of the equal sign to the other and unknown should be always on the left side of the equal sign. That is why the students faced the difficulties in solving the equations having unknown either on right side or as a divisor.

Table 3: Percentage of students adopting strategies in solving equations involving grouping of numerical term

Eq. No.	Equation	C	ISA	SF	RS	G	SS
12	n+34 = 29+38	45	5	0	5	55	0
13	23+n+18 = 44+16	45	5	5	5	50	0
14	4+n-2+5=11+3-5	35	0	5	0	35	5

C=Complementary Subtrahend, ISA=Inverse Subtraction Algorithm, G=Grouping of Numerical Terms, SS=Systematic Substitution, SF=Substitution on First Trial

Majority of the students solved the equations involving the grouping of numerical term through complementary subtrahend procedure (C) and grouping of the numerical terms procedure (G), while very few used Inverse subtraction algorithm (ISA), and random Substitution (RS). Of the students who adopted the grouping the numerical terms (G) with complementary subtrahend (C) strategies, majority of the students successfully solved the equations. It indicates that students have difficulty with transformations.

Table 4: Percentage of students adopting strategies in solving equations involving additive & multiplicative operations

S. No.	Equation	C	SF	G
15	13n +196 =391	25	5	5
16	420 = 13n + 147	25	0	0
17	16n – 215 = 265	25	0	0
18	63 – 5n = 28	20	0	0
19	188 = 15n - 67	25	0	0

C=Complementary Subtrahend, G=Grouping of Numerical Terms, SF=Substitution on First Trial

Majority of the students used complementary subtrahend/factor procedure to solve equations involving additive and multiplicative operations. Students who adopted the complementary subtrahend/factor strategy, successfully solved the equations but on the other hand a large number of students could not solve the equations because they only knew that they had to either subtract/add or and divide the numbers. They were not operating on the unknown.

Table 5: Percentage of students adopting strategies in solving the equations involving double occurrence of unknown on the same side

Eq.No.	Equation	C	SF	GU	SS	S +Ap
20	n + 5 + n = 55	25	10	25	5	5
21	11n + 14n = 175	30	0	30	0	0
22	9n – 4n = 35	35	0	35	0	0
23	7n + 5n + 7 = 55	45	0	45	0	0

C=Complementary Subtrahend/Factor, GU=Grouping of Unknown Terms, S+Ap= Substitution and Approximation

Majority of the students used complementary subtrahend/factor procedure (C) and grouping of the unknown numbers (GU). Some of the students also used substitution on First Trial (SF), Systematic Substitution (SS) and Substitution and Approximation (SAp) Procedure. Of the students who adopted the grouping the unknown (GU) with complementary subtrahend/factor strategy, almost all the students successfully solved the equations. But at the same time, a significant number of students could not solve the equations because they only knew that they had to either subtract/add or and divide the numbers.

Table 6: % of students adopting strategies to solve the equation with unknown on both sides

S.No	Equation	C	GU
24	5n + 12 = 3n + 24	19	19

Only half of the students solved the equation involving double occurrence of unknown on both sides through complementary subtrahend/factor procedure (C) and grouping of the unknown numbers (GU). Of the students who successfully solved the equation, majority of them used complementary subtrahend/factor strategy(C) and grouping of the unknown numbers (GU).

Conclusion

The findings reveal that the children have procedural knowledge to solve the equations and the most adopted strategy is complementary subtrahend/ factor procedure. The strategies used by nearly all students varied from equation to equation. Most students chose a simple strategy that was sufficient for solving a particular equation, rather than necessarily using the most sophisticated strategy they were capable of. Not surprisingly, one-step equations were easier than those involving two or more steps. However this study has shown that the strategy of solving one-step equations by inverse operations is used by more able students than those who use either known basic facts or counting strategies. These strategies in turn were used by more able students than those who solved one-step equations using guess and check. Inverse operations are clearly required for the strategy of working backwards on two-step equations. It is important for teachers to realize that success at solving one-step equations does not necessarily mean that students can use inverse operations or that those students are ready to attempt two-step equations. Of particular note was the finding that using an inverse operation to solve an equation involving division was very difficult for students. The strategy of transformations was used by only the most able students, with many students reverting to guess and check for equations with unknowns on both sides. As has been clearly identified (Herscovics & Linchevski, 1994), students have great difficulty with transformations.

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Rethinking Primary School Mathematics: Directions for a Process-Oriented Curriculum

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"In the question lies the answer" **-**Raja P.K.

Introduction

In recent years, the status of mathematics education in India has drawn considerable attention, both in scholarly discussions and popular conversations. Learning outcomes in mathematics at the primary and secondary levels have brought to surface a series of concerns. For instance, the successive Annual Status of Education Report (ASER) surveys conducted at the national level have shown primary and secondary school children's learning achievements in mathematics and language to be below their age-appropriate standards (ASER, 2011). Documenting a steady decline in 'mathematical ability', the ASER study further showed that only 66 per cent of children in Std I could recognize numbers from 1-9; only 37 per cent of Std III children were able to do two-digit subtraction; and 36 per cent of Std V children could perform basic division (ibid, p.52). Other studies has similarly confirmed these trends.

It has been found that early grade outcomes and difficulties are linked to later achievements in mathematics and can become obstacles for development of higher mathematical concepts (Jordan, Kaplan, Ramineni, & Locuniak, 2009). A testimony to this fact is perhaps the results on the 'Trends in International Mathematics and Science' (TIMSS) survey which showed that 42 per cent of high school children from Rajasthan and 50 percent from Orissa, failed to meet the international low-bench mark for mathematical knowledge (Das & Zajonc, 2009, p.3; Kingdon, 2007, p.5). This failure becomes more significant when seen in the context of questions of social equity and justice, since mathematical literacy is a prerequisite to deal with socio-political and economic structure of society (Frankenstein, 2010). It also regulates access to scientific-technical jobs. The lack of ability in handling and decoding mathematical information can thus reduce democratic control over social realities (ibid) and lead to reproduction along caste/class lines.

While several factors such as school quality, infrastructure, teacher education, pedagogic practices, sociocultural factors such as parental education levels, and so on, have been proposed to explain these poor outcomes and achievements in mathematics, in this paper I particularly draw attention to the role of the 'textbook culture' (Sarangapani, 2003, p. 124) and the 'strongly framed curriculum' in sustaining the difficulties children experience with mathematics. My own interest in the role of the textbook and the curriculum emerged in the course of my work with primary school children in a remedial teaching set-up[1]. Here, it was seen that children with apparently age-appropriate levels of cognitive maturity, having no Special Learning Disabilities[2], belonging to well-to-do middle class families with educated parents, attending good quality private schools with no major structural deficits were also struggling with primary level mathematics. Further it did not appear to be that the children lacked in mathematical concepts per se that affected their performance on mathematical problems (an observation that was similarly made by Russell and Ginsberg, 1984 in their study as well). It was in this context of understanding the nature of mathematical difficulties that these children experienced, that the role of the textbook and the curriculum as a contributing factor to the difficulties became prominent. An attempt is made in the following sections to further elaborate on this by examining what happens to children's intuitive mathematical learning when it encounters the formal curriculum and the text-book, which has a 'regulating effect' on classroom transactions and controls the 'products of learning' (Sarangapani, 2003, p. 124). In the final section, directions for minimizing these obstacles and difficulties that children face in the mathematics classroom is developed, by drawing on the 'pedagogical content knowledge' (Shulman, 1986) that I applied in the remedial classroom. Based on this experience, it appears that developing a process-oriented curriculum may be helpful in developing mathematics as a way of thinking, interpreting and acting on the world, rather than as a set of procedures and formulas. This can be useful in reducing difficulties that children experience with mathematics and contribute to democratic control over mathematics learning.

[1] I would like to acknowledges the support and opportunities provided by Parijma Medical Centre, Bangalore, where I worked within the remedial teaching set-up and drew insights on teaching-learning practices and the curriculum that is presented in this paper.

[2] Children were assessed using standard psychometric tests of intelligence and learning difficulties before they were recommended for remedial learning classes.

The Relation between Early Mathematical Abilities, Text-book Culture, and 'Habits of the mind'[3]

Studies on children's mathematical learning show that abilities such as quantification are present early on in infancy itself (Vilette, 2002), laying the basis for later learning. Several other studies also point out that numerical reasoning and numerical transformation abilities are fairly well developed even as early as preschool (Gelman & Gallistel, 1978; Hughes, 1986). Hughes (1986) reported that 3-5 year olds with no prior formal instruction in addition and subtraction were successful in object-represented addition with small numbers, while a small minority was also able to solve addition and subtraction using large numbers (five and above). Further, he found that the rate of success on addition and subtraction for hypothetical objects and formally represented sums in mathematical language was significantly lower than for object-represented sums. Other studies also show children to be capable of some mathematical reasoning and inferences even without explicit instruction (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998). For example, the study by Hartnett and Gelman (1998) showed that even beginning school children are able to produce the implicit successor principle for natural numbers when they have mastered rote number learning to some extent and this learning is scaffolded by clues to look for systematic patterns of the form {N+1 > N}.

Other evidence for children's intuitive understanding of implicit principles in mathematics was seen, for example, in a study on the inverse relation between addition and subtraction. Lipton and Spelke (2006) found that children who could not count beyond 60, were still able to utilize the inverse principle for equations of the form of {x+y-y=x} for large non-symbolic numerosities involving addition or subtraction of 1. Gilmore and Spelke (2008) have further demonstrated that preschool children were able to perform successive operations of addition and subtraction on large non-symbolic <u>approximate</u> numerosities, but were unable to perform successive operations on <u>exact</u>, large symbolic numerosities. They also showed that children appreciate the inverse relations between addition and subtraction for problems involving <u>approximate</u> large symbolic numerosities, but not for problems involving <u>exact</u> large symbolic numerosities.

Other studies on early mathematical learning, such as that by Carpenter et al. (1998), have also shown that children bring their own, multiple, action-based, intuitive problem-solving strategies, to a single class of problems such as addition, subtraction, multiplication or division. This seems to suggest that children's own problem solving strategies differ greatly from the manner in which formal mathematical lessons are planned. For example, in their study, Rachel, a first grade student, used three different strategies for a set of problems involving subtraction: producing a 'take away' or removal strategy for the first; adding to find the difference between two quantities for the second; and using a comparison strategy for the third. Rachel appeared to directly model each given problem in order to find a suitable strategy for them. Similarly, in the case of division, she modeled two problems differently based on the actions emphasized in the problem: thus, she divided the total number of objects given into the specified number of groups and then counted the number of objects in each group for one problem, while she placed a given number of objects in one set and counting the number of sets she made by placing the objects thus, for another. In place of a single formula or procedure that is taught for each class of operations in the formal mathematics classroom, the study showed that, like Rachel, children have multiple heuristic strategies based on actions to solve a range of problems.

While the space here does not permit a review of more such studies in order to elaborate on children's early mathematical learning prior to the onset of formal instruction, the few illustrations presented should suffice to draw attention to some fundamental insights for planning primary school mathematics education. First, the studies have made it amply apparent that children show natural ability to seek mathematical patterns, visualize outcomes and hypothesize, which are perhaps inherent 'habits' of organization of the human mind (Cuoco, Goldenberg, & Mark, 1996). Recognition and explanations for apparently implicit concepts such as the successor principle, and the ability to evaluate the outcomes of approximate numerosities even before developing a language for these numbers, seems to show that children actively seek to construct their own mathematical meaning, a view that is consistent with the constructivist school of thought.

Second, echoing Piaget, several of the studies seem to indicate that there is a natural progression from concrete to the more abstract in children's thought. This is also reflected in the findings of Carpenter et al. (1998) that there is a shift from direct-modeling strategies to counting strategies (which reflect a continuation of the action-based strategy on symbolic entities), and finally to derived-strategies in children's understanding and use of mathematical operations. These findings may also have important implications for planning mathematics lessons, since children's early understanding of concepts such as the 'successor principle' and 'inverse relations', may be based on concrete experiences rather than on an understanding of abstract logical rules. The importance of paying attention to this was emphasize by Piaget (1967), who argued that children younger than 6-7 years do not understand 'true reversibility' that is based on logical operativity, but rather function

[3] The subtitle 'Habits of the mind' is drawn from Cuoco, Goldenberg and Mark's (1996) paper Habits of Mind: An Organizing Principle for Mathematics Curricula, that provides a vision and important directions for framing the mathematics curriculum. Here, the title is borrowed in an attempt to analyze the early occurrences and influences of the mind's organization principles through studies of early mathematical reasoning

with 'empirical reversibility' which is based on the juxtaposition of two different motor actions or perceived events that appear reverse of each other (as cited in Vilette, 2002).

This, together with the earlier studies which showed children's difficulties with symbolic representations even after attaining mastery over non-symbolic problems, seems to indicate that the difficulty with mathematics that children experience, may arise as a result of an 'incomplete' or 'inappropriate'[4] understanding (Ryan & Williams, 2007). The difficulty in undertaking this transition from empirical to abstract thought may perhaps be the underlying cause of their difficulties rather than the absence of concept per se. For example, Ryan and Williams (2007, p.18) provide an example of how 'incompleteness' in understanding may affect mathematics learning and performance with the example of fractions: while an empirically derived understanding can aid solution on problems such as how to divide 6 cakes among 12 children, or how may half cakes make 6 cakes, the same empirical strategy cannot be extended to solve a sum such as {6 / ½}. In this case, children's intuitively developed empirical reasoning may not aid their performance, thus leading to obstacles since the logical understanding of fractions has not been developed.

Further, 'incompleteness' in understanding can affect outcomes by limiting the flexible use of concepts and increasing dependence on 'primitive' inefficient strategies (Ryan & Williams, 2007, p.57). Such strategies may increase the cognitive load, thereby increasing the chances of errors too. For example, applying the long multiplication approach to a problem of the nature '7X99=?' may demand greater executive and working memory resources in comparison to the application of a distributive strategy such as 7 × 100 – 7 (ibid). The ability to substitute strategies would, however, require an understanding of the true inverse relations between subtraction and addition. In the context of my own work too, use of such inefficient strategies, leading to errors were also noticed. For example, for the sum given below,

$$\begin{array}{r} 4 \\ +7 \\ \hline \end{array}$$

children often preserved the order in which addition was carried out, without recognizing the commutative property of addition. Therefore, children count from the first given number (4), upwards till 7 more is reached (i.e., "after 4, five, six, seven…eleven"). This strategy proved to be inefficient for two reasons: first, children had to count more numbers (7 rather than 4) and therefore required more cognitive resources of attention and memory; second they had to cross seven on the mental numberline and continue after seven, without confusing the cardinal property of 7 with its ordinal position to reach 11.

In addition to 'incomplete understanding', 'inappropriateness' in understanding also creates its own set of problems, leading to errors and difficulties in children's work. An illustration of how 'inappropriateness', i.e., arriving at a different set of conclusions from what is intended within the formal mathematics lesson, affect outcomes has been presented by Baroody and Ginsberg (1983) in their study on children's understanding of the '=' ('equals') sign. Children appeared to interpret the sign as an 'action performed', i.e., they view it as part of the form that a particular mathematics equation takes (e.g., 5+3=8), rather than as a symbol that stands for "the same as". Therefore, children reject or have great difficulty in solving problems with unconventional presentations such as {? = 5 + 8; 13 = 5 +?; 13 = ? + 8}. The authors further reported that this restricted understanding of "equals" persists through elementary school, and many times continues through high school and college, thereby affecting later learning as well.

Similarly, within the remedial training set up, evidence for 'inappropriate' understanding of principles or concepts, and their effects on outcomes was amply visible. For example, it was seen that children had great difficulty performing word problems that did not present familiar terminology, for example, 'total' or 'all' for addition, or 'remaining' or 'left' for subtraction. In situations where the statements did not follow the conventional format, as given below,

> "A basket contains 15 mangoes. 5 rotten mangoes are removed from the basket. What is the total number of mangoes in the basket now?"

children showed confusion in taking decisions about the operations to be performed, since the terminology was misleading (i.e., the use of 'total' which was associated with addition, were used as cues therefore leading to errors). These instances demonstrated amply that the children had developed a different set of associations than what had been intended from previous experience with these problems.

Thus, in this manner, the inappropriate or incomplete understanding may limit the smooth transition from concrete, empirical reasoning to abstract, logical reasoning, unless children are provided a cues and opportunities to explore these connections. Formal learning often fails to bridge this gap by providing opportunities to explore and develop a related set of meanings for concepts (for e.g., by allowing children to examine the commutative property of addition during an addition lesson; exploring the related sets of actions that subtraction may involve such as making comparisons such as 'how much more/less than x?', finding the difference between two quantities, removing or taking

[4] A qualification of the term 'inappropriate' must be duly made here since the authors do not use it to mean a child-centred deficiency in cognitive ability. Rather the term is used in their book to highlight the manner in which the child's conception may differ from intended formal 'mathematical way' or lesson, or from what the teacher or the assessor has in mind and what is demanded in the academic or school context.

away, and so on; examining alternate definitions for even numbers such as 'numbers divisible into "fair shares", 'numbers divisible into pairs', alternate numbers on the numberline, any large number whose unit digit is divisible by 2, Ball & Bass, 2000; and so on). While the earlier example about Rachel shows that children naturally show an inclination to bring these inter-related meanings to mathematical problems, they may not conceptually relate them together in order develop a complete understanding of the concept, unless provided the scaffold to do so.

Therefore, what these examples seem to point to is that the formal and procedural manner in which these concepts and principles get introduced to children within the mathematics classroom, and the manner in which they get 'reinforced' through 'repeated workbook exercises' (Baroody & Ginsberg, 1983), do not always sufficiently close the gap between the abstract logical nature of mathematics and the empirical, intuitive meanings that children develop. Boaler (1998) has also pointed out similarly that students who underwent a traditional mathematics course that was text-book governed learnt procedural knowledge while students who underwent a project-based course were '"apprenticed" into a system of thinking and using mathematics that helped them in both school and nonschool settings. The 'errors' or difficulties that children demonstrate, thus seem to arise because the mathematical principles that "humans naturally develop … informally and intuitively in 'non-mathematical', prototypical ways" seem to conflict with formal learning, as pointed out by Ryan and Williams (2007, p. 20).

The difficulties caused by formal mathematics learning involving 'repeated workbook exercises' is particularly relevant in the Indian context, which is dominated by a 'text-book culture'. Here, text-books do not function merely as a measure of practice or habituation, but have a central, overarching effect on the learning situation, displacing the agency of the learner and the teacher, through the production of 'ought-to-know' knowledge that must be digested in the form of a set of 'right answers' (Sarangapani, 2003, p. 141). This knowledge is further marked by 'strong classification' that prevents formation of inter-related meanings by maintenance of strict boundaries between categories and concepts (Sriprakash, 2010). A sample evaluation of the NCERT primary school mathematics textbooks immediately reveals this strong classification, where lessons on topics such as addition and subtraction are sequentially planned with no linkages provided between related concepts (e.g., the inverse relation between addition and subtraction, multiplication as repeated addition, the relation between the cardinality of numbers and addition, and so on). Further the text-book privileges a 'performance-oriented' pedagogy (ibid) by planning and measuring learning as an exercise in 'repetitive writing' or repeated written practice based on the large number of 'copy-cat'[5] problems given under each topic. Further, learning is measured through standardized, explicit evaluation criteria that measure 'performance' (ibid) in the form of 'right answers'. A analysis of the notebooks or 'copies' (Sarangapani, 2003) of children who came to the remedial class similarly showed that the topics had been sequentially covered. Interaction with the children revealed that no space had been devoted either in the classroom or in the books to explore mathematics as an inter-related set of concepts (which also include some implicit procedures and principles). Notebook corrections showed that importance was paid to 'steps' or procedures and the final answer, and not to the process of reasoning that children brought to the problem. Thus, mathematical learning appeared to be *ritualized*, despite the introduction of child-centred pedagogies such as *Nali Kali*, as demonstrated by Sriprakash (2010). Activity-based, child-centred pedagogies that were introduced to facilitate a democratic, participatory and non-authoritarian pedagogy of learning were rather seen by teachers as ways to provide room for affective expression of learners, rather than tools to facilitate student-directed learning to bridge the gap between their intuitive and formal knowledge.

Sriprakash's (2010) work importantly demonstrates that the shift in pedagogic practices alone cannot be of much use when it is still practiced within the context of the text-book culture and performance-oriented curriculum. Curriculum, as the site of intervention, becomes important in this context as the curriculum governs all that is "planned, implemented, taught, learned, evaluated and researched in schools at all levels of education" (McKernan, 2008, p. 4).

The National Curriculum Framework (NCF, 2005) as well as by the National Focus Group on Teaching Mathematics (NCERT, 2006) have characterized the present curriculum as 'tall and spindly', since it structures the syllabus into linear, independent units that build on skills and competencies thought to have already been mastered in the previous grades (Banerjee, 2000). This practice is contrary to research which suggests that children's mathematical knowledge develop gradually through rich elaboration of connections derived from experience, which may not accompany the initial introduction of the concept. Further, this 'outcome-based' model of the curriculum that plans learning as 'units' with 'exit outcomes' that provide the cue to build new concepts on top of the existing knowledge affects learning in several ways as pointed out by McKernan (2008): first, it destroys the epistemological basis of the subject by breaking it down into micro-units, lessons or objectives; second, it trivializes learning outcomes by privileging low-level recall skills; third, it introduces a management orthodoxy that allows room for strong 'framing' and 'classification' privileging 'ought-to-know' knowledge in the form of right answers; and finally, it constraints learning by preventing the use of

[5] Problems are typically presented in a repetitive manner with little variations in format – for example, addition word problems using the terminology of 'total' or 'all', as mentioned earlier; equations having the same syntactic structure of x + y = ?, instead of exploring alternate structures such as x + ? = y, and so on.

spontaneously arising unanticipated opportunities of inquiry and thereby reducing learner agency and control over learning in the classroom. These effects of the outcome-based model can thus create the many difficulties that children experience with formal learning, particularly in subjects such as mathematics.

'Learning from Errors'[6]: *Restructuring Curriculum as a Process*

Following Ryan and Williams (2007), the earlier discussion of children's errors in understanding mathematics can provide a useful entry point in developing a curriculum that can address children's failure or poor outcomes in mathematics. However, as the authors point out, 're-presenting' a 'failing'[7] curriculum cannot be the way forward with the 'failing' child, without identifying first the failures of the curriculum. It is important to remember that children 'reinvent knowledge' (Freire, 1973) by acting on their environment to construct meaning. This therefore, suggests that learning cannot be seen as a 'finished product' that can be deposited (ibid) with children's minds. Rather, it involves a process of ongoing meaning making (Lave, 1996), that arises through the continual process of action and reflection (Freire, 1973). This challenges the view that knowledge is neutral, objective and accessible to all as suggested by the formal classroom discourse, since there is no artificial dichotomy between the object of learning and the subjective learner. If knowledge arises through action and reflection on our own experiences, then class, caste and other sociocultural backgrounds play a powerful role in making meaning of classroom discourse. Thus, when intuitive and empirically derived knowledge shaped by one's social background clashes with formal classroom discourse, it gives rise to an anxiety among learners to 'get at' the intended meaning. The lack of access (due to social differences) and support (due to the manner in which formal learning is designed) pushes students to take recourse in ritualistic performance or what Skemp (1976) has called 'instrumental' learning. Instrumental learning involves learning to apply rules without knowing 'why', thus creating a form of rote learning that increases the cognitive load. This may result in both poor motivation levels as well as high anxiety that can lead to "blind panic or rushing", both of which can increase the chance of errors (Ryan & Williams, 2007, p. 14). Instrumental learning also does not allow children to develop connections and relations based on their own prior experience and learning, therefore, leading to anxieties and difficulties and reducing democratic control over mathematics learning.

In contrast to this method, mathematical learning needs to involve the development of mathematics as a tool of inquiry to operate on the world, through reformulation of the curriculum as a process. In Ormell's terms, "understanding X [in mathematics] consists in knowing how X relates to, or connects with, other aspects of the world" (as cited in Porteous, 2008). Therefore, it involves the process of developing 'relations' or a relational understanding of mathematics, which involves knowing not only how to apply mathematics rules, but also why they are applied (Skemp, 1976). Haylock (2010, p. 25-26) has similarly pointed out that mathematics learning consists of identifying a small "network of connections" that underlies the different concepts. Emphasizing the processual nature of this, Carpenter et al., (1998) have pointed out that this "taxonomy of relationships" need to be developed by children through their own empirical efforts.

Planning the curriculum as a dialectical process of action and reflection (Freire, 1973) to discover the interrelations and the implicit connections between mathematical phenomena may aid the formation of this 'taxonomy of relationships' in children's minds. This requires the development of a process-oriented curriculum that keeps lessons open-ended, allows room for learner-initiated inquiry, focusing on developing the procedures and criteria that embody the discipline rather than specific content (McKernan, 2007). Further, such a curriculum allows room and values unanticipated and creative responses and outcomes (ibid). This could provide teachers opportunities for identifying the 'incompleteness' or 'inappropriateness' in understanding by casting the teacher in the role of an action-researcher who can draw on his/her 'pedagogical content knowledge' through reflexive practice. Developing this understanding can allow teachers to scaffold children's learning, thereby adequately aiding the transition from empirical to logical-abstract understanding. This process requires developing the curriculum as a discovery of the 'network of connections' in mathematics that will allow scope to flexibly use it as a tool of enquiry: for example, having the first broad topic on numbers explore the relations between numbers as naming devices and their relationship to the cardinality principle; numbers as counting devices and their relation to the successor principle; the relation between how cardinality and ordinality of numbers become related through operations of addition, subtraction, multiplication and division; and so on. In terms of pedagogy, in addition to the activity-based learning methods that have become popular today, two important components needed to be further developed: first, learning needs to be planned as a dialogue where a community of learners question, challenge, defend, and rework their collective learning through an ongoing process of dialogue, negotiation, demonstration, justification and acceptance (Herrenkohl & Mertl, 2009); second learning needs to be developed as praxis (Freire, 1973) by providing opportunities for modeling a series of 'if..then' scenarios through which one's individual conceptions can be reflected upon, reworked and reformulated, and relations with other concepts and the world, can be developed. In this

[6] The idea is drawn from Ryan and Williams (2007) work in which they discuss children's errors in mathematics as a way to organize learning

[7] Experience within the remedial set-up seems to show that the problem of failure may not lie with the child, but in the manner in which the lesson is presented.

process it is also important to bring learners' social and psychological backgrounds to the fore in order to allow them to reflect on their influences on learning. Further the pedagogy and evaluation must involve a process of engaging with the errors' or 'misconceptions' that children demonstrate (Ryan & Williams, 2007), which can have two important advantages: as a pedagogic device, it can help teachers identify the difficulties that children face in bridging the gap between empirical learning and the abstract-logical learning, thereby allowing them to provide the required scaffold to make this transition; as an evaluation device it emphasizes the process of developing meaning and is therefore dynamic and sensitive to children's active efforts at meaning-making, rather than focusing on static all- or- none outcomes that reduce learning to 'instrumental' forms and create an anxiety among children to 'get at' these principles and procedures.

Conclusion

As Ryan and Williams (2007) have succinctly put it "the consequence of not understanding [children's errors] may be repeated re-presentation of a failing curriculum. Understanding such errors can make all the difference to pedagogy and should be prized accordingly" (p.16). This seemed evident from my own experience in teaching as well. The failure of the curriculum mainly seemed to lie in its inability to bridge the gap between children's intuitively developed mathematics knowledge and the abstract, formal nature of the lessons, presented as a set of rules and principles. The learning situation, dominated by the text-book and workbook, focuses on skill development in the absence of conceptual development and measure performance rather than cognition. This wedge in children's learning between the skill and the conceptual understanding underlying it, between performance and cognition of an operation, between action and reflection on it, between empirical and abstract understanding of it, seems to be the underlying basis for the mathematics-related anxieties and difficulties that children demonstrate. In the long run, this wedge will be responsible for the inability to use mathematics as a tool to uncover social realities that are obscured by economic, statistical and technical data, in adulthood and perpetuate and sustain social inequities (Frankenstein, 2010). A process-oriented curriculum can perhaps change this, as well as ease children's difficulties by teaching them to use mathematics as a tool for inquiry.

Acknowledgements

I would also like to thank all my colleagues at Parijma Medical Centre and children and parents with whom I have worked, particularly Master Balaji and his family, for working with them has helped me in my work in remedial teaching as well as has helped me develop insights on curriculum and pedagogy.

I would like to express my sincere gratitude to Dr. Shivali Tukdeo, (Assistant Professor, Education Programme, School of Social Sciences) and Naresh Keerthi (PhD. Scholar), at National Institute of Advanced Studies, Bangalore, for their valuable feedback on this paper.

I would also like to thank Mr. Raja P.K., my husband, from whom I benefited tremendously by discussing my work on remedial teaching and gained several insights. I would also like to thank Mrs. Usha Ravikumar, who helped me with proof-reading and finalizing the paper.

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Understanding Assessment for Learning: Reflective Practice in a Primary Mathematics Classroom

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Introduction

Assessment has always remained an integral part of our education system though the understanding of assessing classroom learning has, in the constructivist curricular framework, called for a shift from assessment *of* learning to assessment *for* learning. The still prevalent conventional notion of assessment is of 'measurement' of learning and serves the purpose of ranking, selection and certification. Stiggins (2005) argues that assessment *of* learning operates on the assumptions that 'anxiety maximizes learning' and 'competition provides motivation' and thus puts the child to a rigorous testing regime of traditional formative and summative assessments. This view of assessment reduces assessment to an event, the child to an object and learning to a product. The counter movement of assessment *for* learning (Black and Wiliam, 1998) critiques this behaviouristic view and places assessment within the constructivist paradigm where assessment is a process in itself. The new approach to assessment thus emphasizes promoting learning by giving agency to the child thereby inducing a feeling of 'want to learn' and being 'able to learn' (Stiggins, 2005). In contrast to assessment *of* learning, assessment *for* learning recognizes the child as a potential user of her own assessment information through 'metalearning' about how she learns and how she can reach a targeted level from where she is at present. As 'promoting learning' (Assessment Reform Group, 1999; Shepard, 2000; Crooks, 2001) in contrast to 'measuring learning' is the focus, assessment *for* learning is designed to become an integral a part of the teaching-learning processes where the rigid boundaries between teaching and assessment dissolve and a new fluid interaction between the two emerges. Here both are contributors to rather than shapers of each other.

Such assessment becomes all the more significant in mathematics, which serves as a killer subject where most children tend to fail to provide the expected 'product' in routine tests, while the present national curricular aims hope to focus more on mathematization of a child's thought, to help her pursue assumptions to their logical conclusions (NCF, 2005: 42). Achievement of these aims however demands not only the teaching of mathematics to be more process oriented with a focus on thinking and reasoning but also an understanding of assessment that matches this form of teaching.

Traditional view of assessment

In a study in a few schools of Delhi (Papneja, 2008) it was found that the majority of examination questions were devoid of a context and emphasised only on procedural knowledge as opposed to conceptual understanding and clarity. Questions are generally included on what teachers find important in the content in contrast to emphasising understanding of each concept, and often repetition of the same types of questions can be seen in the school examinations.

Some typical examples from question papers in primary mathematics are:

1) Add: 235+184
2) Find the LCM of 15, 18 and 24
3) Subtract: 6/7- 9/11
4) Round off the number to the nearest of hundreds
   549, 582, 642, 695
   And so on…

This process of test taking is repeated thrice a year, generally in the months of July - August; November-December as end term examinations and then in a final examination conducted in the month of February - March. Finally Students are graded as per their performance in these tests and the process ends here.

Assessment procedures of the selected school under study

A school session is divided into two terms. In each term a number of written tests are given called formative (FA) and summative assessments (SA). A formative assessment test is given after the completion of a concept (for example one FA after completion of the concept of number, another after completion of place value and so on…) and summative assessment test is given after the completion of 2-3 concepts (for example, number and place value). There is a difference in the formative and summative assessments. In class 4 and 5 and above, the dates of summative are decided by the teacher and students collectively through consensus. This makes the student an active participant in the decision making process of her own assessment. Also, teachers emphasised that in formative tests, questions based on every sub-concept is asked so that the concept as a whole is given importance and attention rather than only a few dominant notions of it.

In all, there are a number of formative and summative assessments in each term decided by teachers as per the requirement and the number of concepts taught. Other than these, there is also a practice of giving one or two question as class assignment after the completion of a sub-concept, say, on number-names. These are called 'informal FA's', taken to have an immediate feedback on the teaching and to allow required changes in instruction.

An end of the term report of each child's performance is calculated by taking the performance of 40% of all formatives and 60% of all summative tests. Teachers revealed that formatives are meant to help the child identify her weak areas and work upon them. This process is supported by discussing and giving a copy of a rubric (a detailed description of levels of understanding and indicators of performance at each level) to each child. So, the major weightage is given to their improved performance by considering 60% of the summative. There is no final examination. End of a year report is made by collating 50% of the grades in each term. This end of year report also gives a detailed feedback about the child as a whole by reflecting upon her strengths and weaknesses, her interests, passion, attitude towards other class mates etc.

An attempt is made in this paper to understand assessment as undertaken in this selected school of Delhi which deviates from the traditional method of assessment and makes it a more rigorous and a reflective process. The school does not prescribe the NCERT textbooks but does claim to follow the NCERT syllabus.

A qualitative study of the assessment system of this school has been carried out using the following procedures:

a) Observations of Mathematics classrooms in standard 3 and 4.

b) Semi-structured interviews with primary mathematics teachers.

c) Discussion with students.

d) Analysis of documents like summative and formative question papers and answer sheets of students, student's notebooks, teacher's log book and rubrics and,

e) Analysis of the process of planning and teachers' discussion meetings.

As enhancing learning is the focus of assessment *for* learning, Black and Wiliam (1998) suggest that the process of assessment has to be made on-going by integrating it into regular classroom practices. "This everyday classroom assessment entails taking the steps necessary to identify the gap between a student's current work and the desired aim - then together figuring out how the gap might be bridged" (Atkin et al., 2005: 1). This makes it necessary to closely observe and analyse the classroom processes to develop an understanding of their assessment practices.

Classroom environment and learning

For any learning to happen effectively the classroom environment should accept the child as an active constructor of knowledge, should respect her exploration and inquiry and must provide a sense of security and comfort for this discovery of knowledge to happen effectively and meaningfully.

In one classroom episode the following interaction was observed:

While discussing the concept of half as a fraction, the teacher drew two rectangles of different sizes on the board. She divided both the rectangles into two with a vertical line and shaded one- half of both. Pointing towards the shaded half of the bigger rectangle she asked - *"what fraction is this shaded part of the whole"?*

Children - "half".

Pointing towards the shaded half of the smaller rectangle she repeated the question, "what fraction is this shaded part of the whole?"

Children - "half"

Teacher - "Are both the halves equal?"

Few children - "yes" (others silent)

Teacher - "How do you get to know that they are equal?"

One child stood up and replied - "because both are halves"

Teacher (looking towards the rest) - "Do you agree"?

Few children - "yes"

The teacher then took a sheet of paper and divided it into two unequal rectangles.

Showing these two rectangles to the class she continued - *"let this bigger part be rectangle A and the smaller one be rectangle B". Then she divided both the rectangles (A and B) into half, keeping it visible for the class. She continued - "this is half of rectangle A and this is half of rectangle B (pointing to the respective ones)", then placing one half over the other she asked- "Are they equal?"*

Children - "No"

Teacher - "why, both of them are halves, then why are they not equal"?

Few shouted - "A *wala bada hai*"

Teacher concluded - "yes, because rectangle A was bigger than rectangle B therefore half of A will also be bigger than half of B".

One child - "ma'am, we can also say that "half of A is full of B" (By chance the size of the two rectangles was such that the statement of this child was true).

Teacher - "this may not happen always but we can always say that if A is bigger than B then half of A will also be bigger than half of B" and the child continued, fascinated by his own observation - "and half of one is full of B"; the discussion continued further with another illustration on the board.

In another instance it was noticed that when after explaining a question on division in class 3, the teacher asked *"aapko samajh aya*?", a sharp resonating "NO" crossed the room which normally does not happen in classrooms that invoke a sense of fear. Thus it can be seen that in these classrooms students are accepted as active learners who construct their own knowledge by constantly exploring and questioning and their questions are respected by teachers as a genuine inquiry. A sense of mutual trust among teachers and students is apparent which provides the comfort and confidence to them to even challenge and argue with the teacher rather than accept their statements as sacrosanct.

The school actively tries to invoke a sense of ownership of space in children by engaging in activities like displaying children's pieces of art in the school compound. A sense of each child being equally important is created not only in classrooms by providing an opportunity to every child to speak at-least once during discussions, but also at the larger level by involving every single child in every celebration of the school, be it a class Christmas celebration or an annual function of the school. This kind of healthy environment of respect and trust is necessary to let the child feel free to express her thinking, to develop her sense of agency, without the fear of being declared "wrong" (Rampal, 2002).

The above interactional episodes also provide us some insights into the nature of questions asked by teachers and students in the primary classrooms and the understanding of mathematics inherent in the school curriculum.

Nature of questions and understanding of mathematics

Black et al. (2003) provide evidences of classroom discussion to be an important contributor in enhancing learning; it needs to be kept in mind that the opportunity to discuss is based on the nature of questions asked in a classroom, which depend on the teacher's understanding of the nature of school mathematics. If a teacher believes mathematics to be a collection of facts and procedures then her questions will demand only factual details in contrast to the teacher who thinks mathematics is a process of reasoning and logical arguments. Her questions will focus more on the how and why of the concept and process involved. In school a considerable difference is found in teachers' understanding of mathematics. In response to the question "what suggestions do you give your students to improve their performance?" some teachers emphasised on 'practice' as a tool to enhance performance and a few stated the need to understand the logic and connection - "*maths man acchha karne ke liye aapko logic samajhne ki jarurat hai*". These views of teachers are also reflected in their classroom interaction, as can be seen in these episodes from two classrooms.

Episode 1

The teacher read the question from the worksheet - "Sharmila has 24 books. She read one half of her books. How many books did she read"?

Children - "ma'am… 12, 12 …"

Teacher - "how do you know?"

Students raised their hands to answer. Teacher pointed towards a girl and continued - "ok, *pehle hum Hitenshi ka method sunenge*". Hitenshi came to the board, drew 24 rectangles and circled 12 out of them.

Other students raised their hands and shouted along – "*hum bataen…!*" One child stood up and said - "ma'am, *main Hitenshi se jaldi kar sakta tha".*

Teacher - "ok, then you explain your method to the class".

Child - "24 ka half 12".

Teacher - "how did you calculate 24 ka half?"

Child - "table"

Teacher - "what in table?"

Child - "12 two ja 24. So half of 24 is 12"

Other child - "plus karke bhi kar sakte hain 12 + 12"

Teacher - "any other method that any one of you has used? No answer, so, can we move to next question?"

Episode 2

Teacher - "What is half of Rs. 30"?

Students (in chorus) - "Rs. 15"

Teacher - "yes, this you can do with your common sense but if you have to find ¼ of 120 toffees then what will you do"?

Pause for a fraction of a second.

"Divide 120 by 4" and she wrote simultaneously on the board 120÷4. "What will you get?"

Students - "30".

Teacher - "yes, 30. What will you do next?" And wrote on the board

30 toffees	30 toffees	30 toffees	30 toffees

"Here what we have done? We have divided 120 toffees into 4 parts. Now how many parts you have to consider - 1, because we have to find one part out of 4, (pointing to ¼ written above). So how many toffees will you get?"

Children - "30 toffees".

The above instances reflect a complex nature of questions asked to transact a concept and the teacher's role in the discussion. Though the questions asked in themselves are closed and do not provide much scope for thinking but the teachers' pedagogical knowledge (Schulman, 1986) in episode 1 played a crucial role in letting children think differently and alternatively. There is not only an emphasised focus on verbalization of one's own how and why while answering - but also on reflecting upon each other's processes which will help children in creating a consolidated conceptual understanding. In the other instance the teacher rejected the child's thinking and way of arriving at the answer as 'common sense' and imposed the 'formal mathematical algorithm'. Such instances in the mathematics classroom send strong messages that doing mathematics is all about applying the right procedure.

Sample questions used in worksheets and tests:

- Solve: a) 569 x 8 b) 78 x 4
- In a Diwali Mela, Rs 40,835 were spent for rent of the place, Rs 15,964 on food items and Rs 9,236 on the decoration. Find out the total money spent for the Diwali Mela?
- Represent 2 thousand and 5 thousand iconically and also show relationship among ones, tens and thousand.
- If each necklace is made using 6 beads, how many necklaces can be made using 24 beads?

  Draw

  Division fact --

  Final statement --
- Solve the following division facts-

  6 ÷ ----- = 3 72 ÷ ----- = ------

  ----- ÷5 = 6 9 ÷ ----- = ------

Interestingly, the school uses conventional assessment questions that are not very different from those found in other schools. The above questions reveal a contrived form of school mathematics, as a collection of mathematical procedures, where doing mathematics is taken as equivalent to applying the right procedure at the right time. This notion is further strengthened by questions in the worksheets such as -

Q. Identify the division problem out of the following and only solve them.

- There are 380 tables in a hall arranged in rows? If there are 5 rows of tables, how many tables are in a row?
- 64 eggs can be packed in one tray. There are 8 such trays. How many eggs are there in all?
- There are 560 trees in an orchard, find the number of trees in each row if there are 4 rows?
- How many packets of 10 biscuits can be made from 630 biscuits?

The questions asked are close ended and do not provide much space for thinking. Either the questions are de-contextualised and demand the mere application of the right algorithm or a contrived view of the context is taken. The questions like Diwali-Mela challenges the daily practices of marketing where in large scale shopping the amount is generally approximated to a nearer unit of hundreds or thousands (as required). Dowling (1998) argues that this type of problem creates a "myth of participation" (believing that one is involved in real world tasks which is not true). The focus seems to be only on the product, as the given spaces to fill in the answer do-not provide much scope for children to write the answer along with the process they used. Even at places where there is space for representing thinking in ways other than in typical symbolic form, for instance by drawing or making icons, the process is made mechanical by giving rigorous drilling exercises. The general thinking of the school seems to be based on the assumptions that 'practice makes perfect' and learning primary mathematics is limited to achieving a pre-requisite base for later learning in 'higher mathematics'.

The school also seems to be abiding by the fallacious assumption that learning can happen in a de-contextualized situation and can be applied as and when needed in real life situations. This assumption is also prevalent in the sequencing of the content in classrooms, starting from the de-contextual problems and proceeding towards word problems. This simple transferability theory is critiqued by authors like Lave (1988) who showed that for applying mathematics learning has to take place *in-situ.* Boaler (1993) argues that as real life is complex and classroom practices cannot replicate it, classroom teaching- learning processes should appreciate the complexity of the real mathematical world, to make learning meaningful and to develop mathematical literates who can understand and confidently function in the world around them (Rampal, 2003a, 2003b)

Setting standards and unfolding expectations before tests

Traditionally schools work hard to hide the questions that will be asked in the examination. Deliberate effort is also made to add questions in which the child make mistakes as they are seen as crucial filtering tools for identifying 'who know what and how much'. The process of designing questions is made a secretive practice to an extent that even a separate examination unit is established in certain schools. The focus remains on identifying what a child does not know and on penalizing her by labelling her with marks or grades or as fail/ pass. This school breaks the notion of 'assessment as secretive' by making the desired learning goals explicit beforehand, as can be seen from these teacher declarations:

"We will have a multiplication FA (formative assessment) on this Friday. The focus will be on multiplication using the lattice method. Keep special focus on zero, where it is placed and what it indicates. You make mistakes while dealing with zero. Class, after multiplying you should write a complete statement and do not forget to add units to your answer. That is must"

"Class 4 you have your computer FA on this 9th Dec. Iss FA

mai hum practical karange , no theory. Aap acchhe se saare keys ke function samajh len - taps key, caps lock, shift, enter key etc. Aap ek baar sari keys ke function revise kar ke dekh len aur nahi samajh aaraha ho toh u can meet me on Wednesday after 12 in computer lab".

In the above instances and in many others of this kind, the teacher explicitly states the expectations to the students by discussing what will be assessed, what they need to focus on, what kind of errors they usually do, and thus makes them aware of the learning goals, which according to Sadler (1989) is an important component of assessment for learning. Stiggins (2005) argues that it will not only motivate the child to engage actively in the process of assessment but also help her to gain an ownership of her own learning. Viewing from a sociological perspective, by unfolding these critical elements of 'how to answer and what to answer' the teacher opens up the space for an equitable engagement to all in the 'pedagogy of power' (Delpit, 2006). This understanding also takes a step forward, towards a notion of assessment for equity, as students are not kept at a disadvantaged position by keeping the norms of engagement hidden.

Using assessment data reflectively

The process of assessment is not limited to stating expectations and test taking. Teachers give detailed feedback to the students on their performance. The data of assessment is recoded by teachers in their log books as shown below.

Name of Child	Marks in Question1 2	Marks in Question 2 19	Marks in Question 3 3=2+1	Marks in Question 4 1
A				
B				

Further a critical analysis of data is done by teachers and the discipline co-ordinators collectively. Thus if in a particular question a majority of students are found to be performing poorly the teachers view it as a feedback on their teaching, and that particular concept or sub- concept is re-discussed in the classroom. For example - appropriate selection of scale for representing numbers in hundred's on the number line was found to be a weaker area in class 3. Though one can question the necessity of representing numbers till hundred on the number line, without any context, and how it can help in enhancing the conceptual understanding of numbers, the area was re-discussed and practice exercises were given. The result of analysis is also used by teachers in planning for their future sessions. Carless (2007) calls this kind of usage of assessment data as a form of pre-emptive formative assessment. For example, they discussed that last year the concept of fraction was not dealt with in class 3. This year class 4 teachers found that students performed poorly in fractions as the entire concept forms a larger chunk and is difficult to understand, considering the time limitation and hence they decided to introduce fractions in class 3.

These assessment data also serve to assess teachers and provide coordinators an opportunity for helping and supporting them. If a particular class as a whole has not performed well, it is discussed with the concerned teacher and her classroom observations are done to identify the reasons and a feedback is given to the teacher to improve on the areas needed. Cosbey et al. (2002) argue that these kinds of practices where colleagues engage in a meaningful discussion is useful for creating and sustaining a community of reflective practitioners and for effective teaching- learning. This kind of practice also reveals that assessment results are not seen as an individual child's effort and poor performance as her inability to do well, but seen as a collective effort of both the child and the teacher (Gipps and Goldstein, 1983). It also breaks the traditional notion of teacher as an assessor and child as the one who is being assessed. Here teacher is both an assessor and one who is being assessed. The child also plays the role of assessor in assessing her own learning with the help of rubrics. Rubrics explain the targeted level of achievements and indicators of reaching that level. Examples are also given that indicate if a child is being able to do a certain type of problem then she belongs to a certain level.

For example, the rubric for multiplication in Class 3 is given as:

Mathematics: Multiplication

Class 3

Name of the child- ______________________________

Objective – To understand the concepts of multiplication (Identification of multiplication problem in different situations and computation using lattice method).

	Level 1	**Level 2**	**Level 3**
Introduction	Identify multiplication as repeated addition. (understanding of 'times')	Identification of array situation in multiplication problem.	Identification of allocation and rate in multiplication problem.
Computation	Computation of 1 digit no. by 1 digit no.	Computation of 1 digit no. by 2 digit no. and 1 digit no. by 3 digit no. using lattice method.	Computation of 2 digit no. by 2 digit no. using lattice method.
Representation	Interprets the answer with proper statement.		

	Type 1- Identify the correct multiplication fact and circle it. 6+6+6+6+6 Is it 5 X 6 or 6 X 5? One packet has 8 pencils, how many pencils are there in 6 such packets? Is it 8 X 6 or 6 X 8? 1+1=2 marks.	Type 1 – In a garden, 24 trees are planted in each row. If there are 9 rows of trees, how many trees are there in the garden? MS- Identification-1 Computation using lattice method-3 Representation-1 (1 + 3 + 1) = 5 marks	Type 1- For a drawing competition, each student gave Rs. 75.how much money will be given by 37 students of class III? MS- Identification-1 mark Computation using lattice method-5 marks Representation-1 mark (1 + 5 + 1)=7 marks

This helps the child to be an assessor of her own learning, as was noted by some children during discussion. Suggestions for improvement are also given in rubrics that may help the child to improve on the required areas. As this study is on-going a further analysis of the nature of rubrics, assigned target level, and evidences of achievement will be done while undertaking further observations.

The log book also contains the checking record of each child that is it indicates if the child's notebook has been evaluated in a timely manner along with feedback given on notebooks. If the child is not giving her/ his notebooks on time, or is consistently making the same kind of mistakes, the teacher talks to the child in person to figure out the issue. The teacher said that within a class of 32-35 it is difficult to remember about every child so this kind of record helps them to have an individual focus on each child.

Grading and feedback

Teachers only give written feedback to students on their assessment sheets though a record of marks is kept by them. According to Doyle (1983), Black et al. (2003) this practice of writing only feedback with no grades/marks positively influences the child as grades/marks distract their attention from the actual feedback to a mechanical comparing of grades.

Few examples of feedback given by the teachers are:

On task of representation of number - 150, 170, 200, 225 on the number line a student chose a scale of 300's and hence all the numbers were seen to fall in the same category.

Teacher's feedback on it -

"*Good attempt but can we choose scale such that all the numbers do not form a cluster within the same category. Relook the method of identifying the appropriate scale in your notebooks. Retry it and show it to me on Monday*"

Similarly, in other instances where children made mistakes, thought provoking questions were asked. For example in the question – "A factory worker earns Rs. 210 as wage for a day. If he was on leave for a day, then how much will he get at the end of the week?"

A child divided 210 by 7.

Comments by the teacher-

Can you get fewer amounts for working more number of days?

Can you think it like – you give Rs.10 to a rickshaw puller daily for coming to school. Sundays are holidays then how much in total you paid for a week?

Though feedback plays a crucial role in enhancing learning, the nature of feedback can also be a subject of critical examination. The above comment to the child– "Can you get fewer amounts for working more number of days?" may even pose a greater challenge of comprehension. Since this feedback is given based on the tentative assumptions about the probable reason for error it may not be helpful for the child, who may not have understood the context and notion of 'wage' for a day or at the end of the week, in the first instance.

Conclusion

Moving beyond the traditional end-of-term examinations and grading, the school can be seen to have made an attempt to make assessment a more learning oriented and reflective process, by actively engaging both children and teachers in the process. Children are made aware of their present level of achievement with the help of rubrics and their journey to the next level of achievement is supported by giving detailed feedback and suggestions for improvement. Along with that assessment data is used effectively by teachers to reflect on their own teaching and that of their colleagues to make required changes in their instruction process and planning. This valuable use of assessment data is necessary not only in enhancing learning but also in creating and sustaining a community of reflective practitioners who actively engage in identifying the probable reasons of poor performance and work upon those.

However, this understanding of the school in making the process of testing (in the form of formative and summative tests) regular and rigorous to enhance learning can be criticised for burdening the child even more frequently. This process can be made more smooth and effective by giving space to other forms of assessment like group projects, portfolio assessment etc. The attempt to involve children in deciding when to assess is appreciable but there is scope for more active and meaningful engagement of the child by providing and creating opportunities for self and peer assessment. Moreover, the school needs to critically reflect upon its shared understanding of the nature of mathematics and the kind of assessment questions to be used to further a higher level of 'mathematiziation' as have been attempted in the present NCERT textbooks.

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Role of Lesh's Model of Multimodal Representation in Learning of Mathematical Concepts

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Mathematical ideas are abstract mental constructs. In order to help students grasp these ideas, they must be represented in a more concrete way using external representations. These external representations take the place of the abstract, mental concepts, and they embody the key properties of the concepts. Since a particular mode of representation cannot embody an abstract concept completely, it is necessary to have more than one representation for each concept. This will stress linkages among different modes of representation, thus deepening understanding. In the past fifty years, there have been several useful analyses of what understanding means in mathematics instruction (Davis,1992; Hart, 1981; Miller & Kandl, 1991; Skemp, 1976; Wong, 1984). Although there is no one single meaning of understanding, these analyses have, nevertheless, highlighted the notion that understanding is intimately linked to connections between know what, know how, and know why, forming a coherent schema in the individual's mind. According to Lesh, Post and Behr (1987), a student "understands" an idea if he or she can:

1. recognise the idea embedded in a variety of qualitatively different representational systems.
2. flexibly manipulate the idea within given representational systems, and
3. accurately translate the idea from one system to another.

Representation is, therefore, a crucial component in the development of mathematical understanding and quantitative thinking. Without it, mathematics would be totally abstract, largely philosophical, and probably inaccessible to the majority of the populace. With it, mathematical ideas can be modelled, important relationships explicated, and understandings fostered through a careful construction and sequencing of appropriate experiences and observations. It is currently held that it is the translation between different representations of mathematical ideas, and the translations between common experience and abstract symbolic representation of those experiences, that make mathematical ideas meaningful for children.

Bruner (1966) suggested three modes of representation – enactive, iconic, and symbolic for modelling mathematical ideas for children. Intuitively these modes suggest a linear temporal ordering: first, enactive; second, iconic; and last, symbols. Lesh (1979) suggested adding two additional modes, spoken language and real world problem situations. Most importantly, he stressed the interactive nature of these various types of representatives, that is, often several modes will exist in a problem solving setting and individuals will routinely reemploy a variety of representation as they reorganize problem components and interrelationships between them.

Objectives of the Study

This present pilot study conducted as part of Project by one of the fourth year student of Bachelor of Elementary Education during her four month internship in Delhi government run MCD primary school from August'10 till November'10 is an effort to explore the effectiveness of Lesh's model of multimodal representation in learning of mathematical concepts. The objectives of the study undertaken are to explore:

Effectiveness of -

a. real Life Situations; Concrete material; Pictorial representation; oral symbols and written symbols in learning of mathematics;
b. effectiveness of the five modes of representation leading to development of conceptual and procedural knowledge.
c. translation from one mode of representation to another mode of representation amongst the individual learners in the class.

Methodology Adopted for the Study

The intern designed her Mathematics Lesson plans to be executed during the internship based on Lesh's model of multimodal representation of mathematical concepts.The concepts undertaken were Division and Fractions. The methodology adopted has been laid down:

- Detailed Content Analysis for each of the concepts to be taught during internship.
- Designing Constructivist Lesson plans based upon Lesh's model.
- Execution of the lesson plans.
- These concepts were taught to grade 4 learners of M.C.D. Jahangirpuri, B Block where the intern conducted her primary school internship as part of her B.El.Ed. curriculum. The effectiveness of these plans was analysed on basis of the learners' responses (oral and written). The students of this school formed the Experimental group for the study. The responses of the students of this group

were also compared with the students of the Control group namely grade 4 learners of M.C.D Rannoula, D-41 taught through the traditional mode. The sample size was 30 in each of the experimental and the control groups. Each of the groups was administered a worksheet each on Division and Fractions.

- Detailed recording and analysis of oral responses of the learners in the experimental group.
- Detailed recording and analysis of written responses of the learners in the experimental group.
- Selection of 5 students from the experimental group on basis of their regularity in the classes and responses to the designed tasks for case studies.
- Questionnaire designed and administered to other fellow interns to analyze the extent they found Lesh's model to be effective in teaching of mathematical concepts, in case, they have employed the model

Analysis of the Data

The data has been analysed on basis of the following:

Analyzing the responses of each of the 5 students (Mohini, Khushboo, Mamta, Saima & Vaishali) selected for case study for each of the concept undertaken on the following aspects:

i) Conceptual Knowledge of the learner;
ii) Procedural Knowledge of the learner;
iii) Mathematical Communication;
iv) Problem Solving;

Citing below few of the selected responses of the students chosen for case study after the students have had been facilitated to learn the concepts underlying Division and Fractions employing varied modes of representations as laid by Lesh's model and an analysis with respect to the development of above stated aspects.

Conceptual Knowledge

Mamta initially was an average child in class. It was observed that she began to extensively employ the pictorial representation for solving the contextual problems. Mamta's response while she explained that 28 divided (employing symbol for division) by 7 =4 and 28 divided by 4 =7 is different by employing three modes contextual situation, written symbols and orally reading the division fact alongside verbalizing her reasoning as (in words of the learner and language employed by the learner):

i) *28 batan hain aur 7 kameez mein lagane hain to har ek kameez mein 4 batan lagaye gaye.*
ii) *28 batan hain aur 4 kameez mein lagane hain to har ek kameez mein 7 batan lagaye gaye.*

Its observed she is able to frequently translate from symbols to contextual situations and exhibiting the meaning associated with division corresponding to equal grouping too which was evident when she pictorially represented the situation.

Mohini when given the contextual situation on chickens and legs namely if there are 32 chicken legs, how many chickens are there? She first drew (pictorial representation) 32 legs and then made groups of 2 legs each corresponding to one chicken. She translated her response into symbolic division fact 32 divided by 2 = 16. Mohini translated from contextual to pictorial and then to symbolic mode.

When confronted with the expression 15 divided by .... = 3 ; it was observed Vaishali was initially confused. But eventually translated the expression in symbolic written form to oral symbolic form and framed an appropriate contextual situation as: There are Rs.15 to be divided equally among children, each child gets Rs.3. So, how many children are there? She could now answer even mentally that there are 5 children exhibiting her conceptual understanding of division as inverse of multiplication and division as a process of equal sharing/grouping.

Procedural Knowledge

Mamta: It was observed that she initially had problems related with the comprehension of the symbol used for division and equal to as well as expressing a division fact. She was not able to differentiate too the difference in the meaning of the two symbols. She still depended a lot on the pictorial representation to reflect and translate the verbal expressions to symbolic expressions. Gradually it was observed there was translation from symbolic to the contextual situation facilitating the comprehension of symbolic division fact and the operation/strategy to be employed to find the requisite product as evident when she translated 52 /4 (52 divided by 4) as 52 rupees to be divided among 4 students of her class. She then employed the strategy of equal sharing pictorially instead algorithm.

She drew 4 faces with distributing each face an amount of Rs 10 each and said Rs 12 are left and then redistributed equally an amount of Rs.3 each.

She expressed the procedure in terms of developmental approach and wrote the quotient as 10+3 and the remainder as 0.

The procedure employed exhibited her relational understanding of division and revisiting the place value concept hence the development of the conceptual structures.

Mohini's response to finding the cost of ¾ kg of potatoes when the cost of a kg of potatoes is Rs.12 as:

She represented Rs.12 as:

* * *

* * *

* * *

* * *

Dividing Rs 12 into four groups equally and said that three groups make Rs 9 so the cost of three fourths of potato is Rs.9/-

Mathematical Communication

Mamta is a shy student who generally is not found to talk about her actions, but when encouraged to speak in class while the study is being undertaken it was found she was able to express well even in mathematical terms. As in case of problem of dividing 56 by 7, she observed that one of the classmate has expressed the quotient as 7 and the remainder as 56, to which she spontaneously responds :

> *Usne bante nahin to 56 vapas aa gaye ,saare ,paise bach gaye to agar 7 ko diye hain to 56 khatam ho jaane chahiye.*

She was thus capable of regulating the process employed.

Saima: It was observed that from beginning Saima is verbalising her actions and making an effort to justify her responses as is evident while finding 1/3 of 18. She constructs a contextual situation as:

> *There are 18 marbles to be divided among three. How many will each get.........*
>
> *Let me divide it among you three...... (Her classmates)*

So she divides and expresses as:

> *18 ke teen hisse kiye jain to ek hisse mein 6 cheeze aayegein*

Problem Solving

It was observed Mamta was interpreting problems and devising her own strategies to solve those problems. For example for solving 12 divided by....... =2, she started with the multiplication table of 2 and when she arrives at the product 24 she gets caught in conflicting situation. She again translates the symbols into contextual problem: *12 rupey hain pata nahin kitne jan hain ki sabko 2 rupey miley*.

She then arrives at the answer 6.

Mohini when asked to respond to :

How many necklace can be made out of 18 beads if each necklace requires 6 beads.

Instead of solving directly by employing an algorithm asserts the following strategy:

> *1 Necklace 6 Manke*
>
> *2 Necklace 6 Manke........hogaye 12 Manke*
>
> *3 Necklace 6 Manke....... ab ho gaye 18 Manke*

And states in symbolic form as 18 divided by 6 is 3

Analysis of the effectiveness of concrete mode of representation with respect to the entire class: In the beginning it was felt that the activities are time consuming and the results not effective. On the first day of teaching division when the beads were provided to the learners they enthusiastically started forming necklaces. It took time to bring their focus from play to concept of equal sharing of beads in the formation of each necklace and once this was facilitated they started working effectively with variety of concrete material provided with an effort to make equal groupings as required. The concrete material to a large extent ensured the involvement of all the learners despite their previous conceptual structures.

A simple activity of dividing 20 sticks between two students leads to observation of varied responses in class between different pairs of learners: a) Learners who divided by giving 1 to each partner alternatively; b) Learners who divided by giving 2 to each partner alternatively; c) Learners who solved even mentally that each will have 10; d) Intern reflected on Vaishali's response that she directly picked up 10 matchsticks and gave those to her friend to share equally. She emphasized on the mental calculation and verified it with use of concrete material.

In the task of finding different fractions of ribbon of length a metre it was observed:

a) Learners folded ribbon into half and then measured each length using the metre tape and established half of metre is 50 cm;
b) They further extended the activity by folding the half again and finding the length of one fourth of a metre as 25 cm;
c) Scaffolding by the teacher encouraged them to find three fourths of a metre and further justifying that the sum of half and one fourth of metre is same as three fourths of a metre.

Analysis of the effectiveness of Real Life Situations representation with respect to the entire class: It was wonderful to see that students generally tend to put themselves in the situations and then devised strategies to divide. During division corresponding to the situations they designed their own strategies as in the contextual situation corresponding to Kameez and Buttons ,a student, devised a strategy as

> *1 kameez.....4 button;*
>
> *2 kameez.....4 aur button........8 button*
>
> ..
>
> ..
>
> ..
>
> *7 kameez.....4 button........28 button*

She translated the comprehension in symbolic form as 28 divided by 4=7

Similar translation was observed when the contextual situation corresponding to finding the number of goats when the number of legs was given was asked.

Thus contextual situations provide framework to comprehend the meaning associated with the operation and also facilitates the translation to pictorial and then to symbolic form.

Analysis of the effectiveness of Pictorial mode of representation with respect to the entire class: A number of tasks including picture stories were provided to learners requiring the learners to write the **division word problem** and also the division fact corresponding to the picture were provided to the learners. For example distribution of given number of ladoos (in pictorial form); lead students to draw boxes surrounding the ladoos and different students dividing in different equal groupings and hence formulating different word problems and division fact and appreciating multiple responses. The pictorial representation facilitated the verbalization of division facts as well as expression of the written symbolic expression in terms of division fact.

In fractions the pictorial mode facilitated to great extent understanding associated with fraction corresponding to dividing into equal parts of the unit region.

The learners also exhibited the variability in dividing the unit region into equal parts as required corresponding to given fraction.

Analysis of the effectiveness of written symbolic mode of representation: The researcher made an effort to introduce and revisit the symbols and symbolic expressions alongside concrete, contextual and pictorial mode. It was observed that this facilitated the comprehension of symbolic expressions and was evident when learners themselves (many of them) were able to write the corresponding symbolic expression for the pictorial and contextual situations. Initially learners were confused as with Lakshmi for contextual situation of dividing 6 bananas equally among three monkeys she expressed the fact as 3 divided by (using symbol for division) 2 divided by (using symbol for division) 6 on the black board. When asked to interpret she shared it as:

> *''3 bandar hain aur 2 kele denge to har ek ko to 6 kele kul honge.''*

The student teacher interrupted and repeated the question knowing Lakshmi knows the answer but was not able to express it symbolically.

It was also observed that learners are confused with symbols for multiplication and division while expressing the fact corresponding to given contextual situation.

Analysis of the effectiveness of oral mode of representation: When students were asked to verbalize they started reflecting on their actions. Example when the intern expressed division facts in symbolic form or as expressed when one tends to employ the algorithm it was observed many learners translated them orally in form of contextual situations. Kashish on confronting 65 divided by 5 as division fact translated 65 into 5 ten rupee notes and left with Rs 15. She even made five faces with each having Rs.10 and now left with Rs15 to be equally distributed among five and hence each face to have Rs 3 more. So 65 divide by 5 gives 13. Oral verbalizing leads to determination of the requisite quotient. While Kashish was performing and asked to justify why each of the person have to be given an additional amount of Rs.3 after distribution of Rs.10 to each. She justified by reading multiplication table. Another student intervened and mentioned that if we give more than 3 it will lead us to "udhaar". But we have to give only Rs.15. This oral expression leads to an in-depth insight to learners understanding.

Analysis of Control and Experimental Group: A worksheet was administered to the students in each group for both division and fractions and the responses compared both qualitatively and quantitatively.

Quantitative Analysis

Mean of the Experimental group in Division 12.88

Mean of the Control group in Division 5.88

Mean of the Experimental group in Fraction 8.88

Mean of the Control Group in Fraction 6.75

Qualitative Analysis

- It was observed that experimental group members were efficient problem solvers and able to abstract the information given in the problems.
- The students in experimental group were applying the algorithm with respect to developmental approach as compared to mechanical approach in control group.
- The experimental group exhibited many instances of translation in their solutions prominently translating from pictorial to symbolic mode.
- The experimental group showed degree of flexibility as they were able to construct meaningful word problems corresponding to given numerical facts.
- Experimental group was found to be able to manipulate the symbols and the symbolic expressions.
- Experimental group showed positive disposition towards the task assigned.

Analysis of the Questionnaire given to other Interns (Student Teachers):

- It was found nearly 100% of the other interns followed Lesh's model and believed in its effectiveness.
- Eighty percent of them mentioned that out of the five modes of representation concrete mode of representation lead to conceptual development the most followed by the pictorial mode.
- Most of them gave classroom instances supporting the conjunction of contextual situation and manipulatives to support the development of relational understanding in mathematics.
- Lesh model facilitates the construction of mathematical concepts.
- Lesh model empowers students to relate classroom experiences with their daily life making learning process a meaningful endeavour.

Conclusions

- In order to make a transition from traditional mode of representation to employment of Lesh's model in class is a little challenging in the beginning.
- Lesh's model encourages learners to reconstruct the mathematical concepts.
- Learners can be encouraged to be involved in the process of abstraction and generalization of mathematical concepts under study via employing Lesh's model;
- Lesh's model provides teachers with a tool to facilitate the development of meaningful mathematical concepts.
- Lesh's model empowers the development of varied mathematical skills laid down in NCF'05.

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Insights and Challenges in the Teaching-Learning of Probability

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This paper is based on the evident gap that exists between the theoretical exhibit of probabilistic content as presented in the textbooks and the daunting realities that reveal a colossal gap between students' lack of understanding, interpreting and connecting the simplistic of probabilistic constructs to their real life experiences. It is rather poignant to say that the way in which our students interpret probabilistic information, which is commonly embedded in day-to-day situations, is indeed unsatisfactory and thus needs attention.

Probability literacy (Gal, 2005) is making informed judgments on uncertainty and to do so we need skills that are far more complicated and profound than throwing a dice or tossing a coin or drawing balls from an urn. School teachers and students often experience a divide in connecting probability concepts (as represented in the textbooks) to the real-life situations involving uncertainty. In this paper I have tried to put forward the issues and challenges that needs rigorous contemplation regarding the content of probability at the secondary school level and thus on the probabilistic understanding that is (ideally) expected from the students.

Probabilistic thinking is multifaceted and develops slowly over time. Most of the studies done so far have tried to look at the relationships between the intuitive thinking that individuals hold with regard to probabilistic situations (Fischbein & Gazit, 1984; Fischbein, 1975) and how a coherence can be established towards a formal, mathematically based solution to the probabilistic situations (Borovcnik & Peard, 1996; Shaughnessy, 1992; Konold, 1989). While such studies contribute towards the extension of understanding the 'formal' concepts of probability, they lack in deliberating on the idiosyncratic knowledge generated as a result of interplay with the experiences of daily life of the learners. Such knowledge negotiates for or against the formal, deterministic knowledge that the school presents.

Through this paper the intent is only to highlight the issues and some areas of gap that exists when students try to comprehend probabilistic content as presented in the textbooks. By taking snippets from the textbooks, students' responses and teacher's pedagogic understanding on some foundational ideas/terms of probability, the paper is an appeal for re-envisioning the content presented in the textbooks. The paper will first give an overview of the curriculum of probability and will then draw attention on three foundational aspects that can rather be seen as impeding blocks in the understanding of probabilistic content:

a) Theoretical divide in approaches of teaching probability
b) Lack of Probability in situ
c) The Binary quantification

Overview of the Syllabus of Probability

In the Indian textbooks, post 2005, the stochastic content is introduced from class VII onwards. In classes VII and VIII both 'Data Handling' and 'Chance' appear under as a single chapter titled 'Data Handling'. The chapter mostly covers statistical topics and introduces learners to the basic terminology related to probability. In these initial stages the emphasis is on introducing probabilistic terms like sample space, events, independent events, etc. From class IX onwards the concepts of Statistic and Probability appear in separate chapters. In IX the chapter of Probability starts with a brief introduction to history and the concepts are introduced through frequency approach that accentuates on conducting repeated experiments, observing the frequency of occurrence of a desired event. For pedagogic experiences it is stressed that 'a large amount of time must be devoted to group and individual activities to motivate the concept; the experiments must be drawn from real-life situations, and from examples used in the chapter on statistics' (NCERT, 2005).

From class X onwards the classical definition of probability is introduced and simple problems on single events, not using set notation are emphasized. In class XI more abstract concepts such as Random Experiments: outcomes, sample spaces (set representation); Events: occurrence of events, 'not', 'and' & 'or' events, Exhaustive Events, Mutually Exclusive events etc have been given. In Class XII the Multiplication Theorem on probability, Conditional probability, Baye's theorem, Random variable and its Probability Distribution, mean and variance of haphazard variable, repeated independent (Bernoulli) trials and Binomial distribution have been introduced. Most of the content in classes X-XII is presented in the axiomatic approach.

Insights and Challenges

Theoretical divide in approaches of teaching probability

Drawing from the historical developments of limited research in probabilistic thinking three approaches of understanding probability emerge: The Classical approach, The Frequentist approach and the Subjective approach. The Classical approach is based on the axiomatic paradigm, the Frequentist approach draws generalizations through experimentation, usually by law of large numbers and the Subjective approach acknowledges the idiosyncratic knowledge and is based on the premise that different descriptions of the same event can give different judgments.

Most of the content under probability chapters in the textbooks is dominated by classical approach and at some, in initial stages, has traces of examples being conducted by empirical methods. Although, the classical theory is the easiest and the most convenient way of teaching probability but unfortunately is least understood by the students when it comes to the application of the content. Similarly, the prototypical experiments mentioned under traditional frequentist approach, such as tossing of coin, throwing a dice, drawing balls from an urn can be easily replicated and performed in the classroom but they miss on the naturalistic intuitions. Albeit some areas of classical and empirical probability may help in learning scientific application of probability, but the understanding of probability for its common use and application is quite distant from these un-natural approaches. It is time to revise the assumptions and give way to natural intuitions, degrees of beliefs and individual interpretations which cannot be bridged by these 'technical ways' of teaching probability. As subjectivity is natural, therefore, a radical shift is required in the teaching of probability which would acknowledge the nascent probabilistic intuitions and counter-intuitions.

Lack of Probability in situ

Probability is 'numbers in context' (Moore, 1990). Probability is a co-agent that helps in understanding the meaning of numbers. However, it is rather poignant to say that much of the probability course in schools is theoretical and has little or no connection to the everyday encounters with uncertainty. The content does not provide enough acumen that can help students face the challenge of decision making outside the textbooks, as presumed in everyday situations dealing with chance and prediction. These observations cause deep consternation on the widened gap between probability 'in books' and probability 'in context'. The constrained view in which probability is dealt in the textbooks will not make students prepared for discerning the situations that they need to be prepared for. The computational aspects create barriers in the conceptual understanding. When dealing with real-life probabilistic situations students use idiosyncratic, informal and critical and non-critical mathematical understanding. The study of probability should not rely only on formulae but must also include a sense of interpretation that is core to the real life situation. Numbers and calculations should act as co-agents in construing the essence of critical thinking and decision making rather than being elusive. Further, teaching probability embedded contextually not only provides explicit understanding of concepts but also implicitly prepares students for a more socially, economically and politically aware person.

The Binary Quantification

Another addend to the concerns of probability is the numerical representation of degree of uncertainty in the conventional sense. It is a common understanding that most information that is present around us and on the basis of which we make decisions may not be in the quantified sense and certainly not in the binary range of 0-1. The basic convention in which probabilistic information is presented around us is usually in percentages or ratios and hardly ever in the binary sense. For example, "there is a 79% chance of rainfall today" or "the most probable hitting rate for a batsman in this inning is 4.8".

It is further noted that most often people interpret the phrases related to probability in different ways that may not even depend on the numerical inputs. Some situations may not even mention the degree of uncertainty in a numerical sense but would, rather, rely on verbal descriptions and still reflect the idea in probabilistic degrees. For example, statements such as 'the chances of girls getting admission to Delhi University, in Commerce stream, are much higher than last year' represent a degree of uncertainty and are, at the same time, open to individual interpretations. Thus, inducing in our students an understanding of non-numerical yet probabilistically profound information should be the key route of understanding probability.

This paper, thus argues to re-view the content of probability that could help students synergise the information permeating around them.

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The Making of An Academic Programme in Modeling and Simulation

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Abstract: While mathematical modeling is not a new enterprise, the ubiquity of computing technologies has made it all-pervasive. One may argue that this is precisely the reason why we need a multi-disciplinary, broad-based academic programme in mathematical modeling and computational simulation (M&S) methods, with a problem-centric approach at its heart.

Designing and implementing such a multi-disciplinary academic programme without precedent in the arena of conventionally stratified academic disciplines can be a challenging task. It requires a strong sense of vision coupled with commitment, endurance coupled with exibility, and team work coupled with wise leadership and organizational skills.

In this paper, we describe the design and development of a Masters-level academic programme in Modeling and Simulation at the Centre for Modeling and Simulation, University of Pune, and first implemented in the academic year 2008-09. The long and arduous process that culminated into this programme began with brainstorming within a core group, a rigorous design exercise, followed by informal expert reviews that led to an initial one-year diploma implementation. Gradual and incremental experience accumulated from running this diploma programme, together with design-redesign cycles, readily available academic and organizational help - and some luck - eventually brought the programme to it current form, i.e., a full-length two year M.Tech. degree programme in modeling and simulation. We also share our outlook and some of the wisdom gained in the process.

Perhaps the single most important take-home message of our exercise is that curriculum design and implementation in any discipline is a serious endeavour that requires commitment on part of the faculty, long-term organizational commitment, will, and support, and a focus on the overall objective by all.

Keywords: *Modeling and Simulation - Computational Science - Problem-Centric Approach - Applied Mathematics, Applied Statistics, and Computing Education - Multidisciplinary Academics - Curriculum Design – Higher Education*

Mathematical Models, Computing, Simulation

To set stage for what we are going to discuss here, let us first consider a few “real-life” problems:

1. At the current epoch, India is getting urbanized at an ever-growing pace. Given the fast growth in privately-owned automobiles, this has invariably led to dense, chaotic, and often dangerous trac, a prime example of which is the city of Pune, India. The design of efficient urban transport systems that maximize human through put while minimizing costs (including technological and environmental) is clearly a need of the time.
2. Since the “liberalization” of the Indian economy, an Indian chocolate manufacturer has the choice, at least in principle, of buying sugar, amongst other raw materials, from around the world. Depending on the state of the world economy and the supply-demand forces, prices of commodities fluctuate over time. Based on past trends in the pricing of the raw materials and the expected demand of chocolates in the near future, the manufacturer needs to make a decision now as to when and where to buy sugar, and how much. Clearly, the manufacturer’s end goal is to maximize profit.
3. The spatio-temporal dynamics of infectious diseases such as bird or swine flu, malaria, rabies, tuberculosis, etc., has profound implications to human societies across the world. Policy frameworks for dealing with disease outbreaks on a mass scale

necessarily require a detailed understanding of this dynamics.

These illustrative examples above (and, say, a range of similarly-spirited examples in Steen, 2000) come from domains that apparently have nothing to do with one another. However, many such real life problems and their reasonable solutions share many common features. First and foremost, there is a need to understand the system, be able to predict its behaviour, and be able to control it, each at an appropriate level of precision. The first two goals are primarily associated with the "science" part of the process, while the third is predominantly in the technological premises. "Understanding" usually implies identifying prominent patterns of behaviour of the system; in other words, discovering or inferring the "laws" that govern the system. Second, in each of these examples, there is a need to go beyond the qualitative, and understand the problem in a quantitative manner. We therefore need to be able to describe patterns in a exible, economical, and quantitative manner. We may also need the ability to formulate descriptions the same system at varying levels of detail and precision. Perhaps the only "language" that allows describing patterns in this fashion and making inferences about them without actually having to observe the system endlessly is what we call *Mathematics*.

The fact that mathematics is being used for solving problems in almost every domain of human activity is not a new revelation; in fact, Bickley (1964), e.g., pointed out some five decades ago that "Mathematics is relentlessly seeping into the very foundations of the civilisation in which we live and the community of which we are part ... ." Moreover, this game of using mathematics for describing and "understanding" patterns is even more ancient, as evidenced by its use in the natural sciences; in fact, we could go as far as saying that mathematics was perhaps *born* way in the depths of time.

A mathematical description of a system under scrutiny, referred to as the *mathematical model* of the system, is built by extracting features of the system that are most relevant to the problem at hand. This is the process of abstraction: the actual system is now replaced with its model which tries to capture the essential features of the system that are most relevant for the problem at hand. Understanding the system with reference to the problem at hand is now equivalent to understanding the mathematical model. Mathematical models therefore need to mimic the system sufficiently accurately, which implies that models need to be validated against reality. Better understanding of the system, arrived at through observation, invariably leads to changes in its mathematical model, some times through generalization of the model to incorporate finer nuances of the system's behaviour, and some times through outright rejection of the current model. This process is perhaps best exemplied by the how mathematical models of the physical world (popularly referred to as theories of physics) changed in the light of new observational evidence. The fact that mathematical models can at best be accurate, but never exact, representations of the reality, is most perceptively expressed in the oft-quoted maxim "Essentially, all models are wrong, but some are useful" and its variant, namely, "Remember that all models are wrong; the practical question is how wrong do they have to be to not be useful." (Box & Draper, 1987; see also Box, 1976).

The mathematical structure of a model gets invariably more and more involved as finer and finer nuances of the system's behaviour get incorporated into the model. This is where pure analytical reasoning become more and more difficult, if not impossible, and greater mathematical sophistication is required to deal with the greater complexity of the model. This is where computation and simulation make themselves indispensable. From a certain perspective, theory of computation itself could be considered a branch of mathematics.

A simulation attempts to represent a model of the real-life system under study using some other well-understood system, the *simulation system*. A natural and versatile choice for a simulation system is the contraption called a computer that performs computation. To quote Bickley (1964) again, "Similar things were happening in the pre-computer age. ... But the computer our powers and widened our scope. In particular, it enables us to conduct mathematical experiments and to construct mathematical models on a vast scale ..." The purpose of a simulation system is to somehow mimic the behaviour of the real-life system under study, and more often than not, this involves the use of a mathematical model of the system. A simulation is thus built on three components, namely, know-how from the problem domain, a (mathematical) modeling formalism, and methodologies and technologies specific to the simulation system.

Another important feature of this problem-centric approach is that we take the view that a problem is a problem in need of a reasonable solution; it doesn't care how we choose to classify it. While traditional knowledge domains have evolved for a good reason into the current organization as sciences, humanities, engineering and technology, etc., the fact that problems arising in disparate knowledge domains could have common mathematical structure necessarily advocates cutting across traditional knowledge domains. For example, what is common to the spread of a disease, the spread of a rebellion, and chemical reaction kinematics? It turns out that commonly-used mathematical models of these systems all turn out to be systems of coupled ordinary or partial differential equations. A breakthrough in one field can thus benefit a completely disparate field because of this underlying similarity of description at the mathematical level.

Why Create An Academic Programme in Modeling & Simulation

While mathematical modeling is conceivably as ancient as mathematics itself, what makes it all-pervasive in modern times - especially the last few decades - is the ubiquity of computing technologies and the ever-increasing availability of comparatively inexpensive computing power. Computation and simulation, in particular, is often seen as a partial substitute for expensive experimentation with the actual system. This has created somewhat specialized niches for people with adequate mathematical background, together with reasonable analytical and computational skills, in almost every area of human activity that benefits from mathematical modeling. Classic examples of this kind include drug discovery, design of automobiles and aircraft, mechanics of engineering structures such as a bridge, pattern discovery in biological sequences, and so on and so forth.

One may argue that this is precisely the reason why we need a multi-disciplinary, broad-based academic programme in mathematical modeling and computational simulation (M&S) methods, with a problem-centric approach at its heart.

The Centre's M.Tech. Programme in Modeling & Simulation

The Centre for Modeling and Simulation, University of Pune, was formally established in mid-2003 with a vision to "promote, support, and facilitate academic and research activities related to mathematical modeling and computational simulation and, in particular, the use of computation as the third scientific methodology (besides theory and experiment)"; "to aggressively promote a problem-centric outlook to real-life problems, and highly multidisciplinary approaches that transcend traditional boundaries separating individual scientific disciplines"; "to keep up with the state-of-the-art in computing and, in particular, develop strong expertise computing technologies such as high-performance computing, grid computing, etc."; and "to create excellent, versatile minds that are capable of learning by themselves, of thinking deeply, of questioning dogma and authority, and of seeing beyond the immediate." (Arjunwadkar et al., 2008).

The Centre became functional with the appointment of its first two faculty members in December 2003. An informal core group consisting of local computational scientists (Arjunwadkar et al., 2005, see History and Credits) in addition to the Centre's faculty became operational almost immediately, and engaged itself in brainstorming about trends in scientific research, engineering, technology, and beyond, with focus on computation, modeling and simulation. Many spirited discussions convinced us that designing broad-based curricula in M&S was not an unreasonable thing to do, even if no such programme existed at that point in time.

The Centre's curriculum design exercise was inspired by the methodical and comprehensive outlook reflected in *Computing Curricula 2001*(The Joint Task Force on Computing Curricula, 2001). Several months of brainstorming and design-redesign cycles eventually led to the creation of the *Advanced Diploma Programme in Modeling and Simulation* (Arjunwadkar et al., 2005). It was a conscious and practical decision to first create a one-year programme so as to get better experience in an unprecedented academic territory, implementation logistics, etc. Experience gathered from running this one-year diploma programme for three consecutive batches (2005-08) made us confident enough to dive into the greater complexities of a full-length (i.e., two-year) academic programme.

The full-length programme in M&S eventually came to be called the *M.Tech. Programme in Modeling and Simulation*(Arjunwadkar et al., 2007). We requested many practising experts to review the programme simply to see where it stands in the eyes of both the academic/research community as well as the industry/corporate sector, and found that it was positively received by all with genuine interest. In fact, we received more positive suggestions than what is logistically possible (e.g., suggestions for courses on topics relevant to specific domains). Most of the feedback thus obtained was incorporated in the structure and curriculum prior to its deployment in the academic year 2008-09. This voluntary review initiative on part of the Centre was over and above the University's formal requirements for approval of an academic programme.

The documents cited above that describe both these programmes are quite detailed and publically available. In the rest of this paper, apart from providing a crisp overview of the M.Tech. programme in Sec. 4, we will therefore try to avoid duplicating the extensive material contained in these documents, and instead focus on salient features of the structure of the full-length programme, design considerations that went into its making, and the lessons learnt in the process. We recommend treating the present article as a companion to these extensive documents.

Structure and Curriculum: An Overview

The M.Tech. Programme in M&S designed and implemented by the Centre for Modeling and Simulation, University of Pune, is full-time 2-year post-Masters (or post-Bachelors in Engineering) programme. The only other pre-requisite for admission in the programme is adequate mathematics background approximately at the first-year level of a typical Indian science Bachelors programme. Core curriculum of the programme is by-and-large focused on methodologies, and is founded on the four principle pillars of applied mathematics, applied statistics, computing, and M&S in practice.

The first three semesters of the programme are devoted to coursework, the fourth to a rigorous full-time project involving hands-on M&S work in any knowledge domain. The training and outlook of the incoming students can vary substantially because of their diverse backgrounds and maturity levels. As such, the first semester of the programme is primarily geared to building a uniform background in essential mathematics and computing. Second and third semesters, in addition to the core courses, also include a selection of elective courses on somewhat specialized topics such as computational uid dynamics, machine learning, etc.

Rationale for the Structure and Curriculum

The rationale for the structure and curriculum could be viewed from three perspectives; namely, (a) a student's perspective, (b) the academic view, and (c) the overall education system. The considerations presented below, which were realized through trial-and-error and continuous introspection, offer some rationale for how our M&S programme came to be in its present form. We present a loose collection of specific insights in the next section.

From a Student's Perspective: The post-Masters level, most prospective students tend to evaluate degree programmes from the point of view of job prospects and value addition. Employment opportunities for a graduate of a M&S programme are likely to come from:

- R&D centres and analysis organizations (in the industrial, defense, academic, government, and other sectors), that use modern computational and simulation methodologies in their design, development, and research/analysis initiatives. Such initiatives include all areas of engineering, science and technology including (but not limited to) materials science and engineering, nanotechnology, bioinformatics and biotechnology, computational uid dynamics, molecular modeling and drug design, process engineering, finance, etc.
- Research- and computing-oriented *support* in R&D or research-and-analysis organizations.
- Research programmes leading to an advanced degree such as Ph.D.

Although some the students may find careers in the IT-related and conventional software industry after undergoing M&S training, that should not be the focus of the programme. However, we do envisage that the skill set and attitude developed through the programme - specifically, problem-solving and programming skills, ability to learn on the fly, and versatility - will enable a student to migrate easily to such careers.

How does this consideration dictate the design and the content of the curriculum? The diversity of problem domains where M&S methodologies can be employed meaningfully argues in favour of a broad-based programme that is primarily focused on methodologies, together with moderate specialization, and exibility to respond to technological advances as well as "market" needs. This is what led to the inclusion of elective courses in our M&S curriculum.

From an Academic Perspective: Independent of M&S considerations, the idea that the academic goals of an academic programme may be characterised in terms of *competence awareness* useful for curriculum design and content. The curriculum content for an M&S programme is therefore dictated by a set of concepts and skills that are central and necessary for *competence* M&S, along with another set of concepts and skills that a student is expected to be *aware*. As mentioned before, a simulation is built on domain expertise,(mathematical) modeling formalisms, and methodologies/technologies specific to the simulation system. Of all the possible combinations of these three components and the two goal levels (competence, awareness), our

curriculum is primarily geared towards training students to be skilled (as opposed to conversant/aware) in the first two components, but be aware of/conversant with a few techniques from the first. This choice was made primarily because all possibilities that require building additional domain expertise into the curriculum quickly become impractical for such a programme.

From a Systems Perspective: The risk of over-generalization, we may say that our education system is organised in the following hierarchy of levels: school, college/university, and the research establishment. Our M&S programme is a part of this whole, and therefore must take into account the inter-dependencies with other levels, as well as with the greater socio-economic reality. This is important because, in our system, failures at one level propagate unhindered to the next. In the context of any programme in higher education, this needs to be accounted for as much as purely academic considerations about the required preparation at the time of entry to a programme such as ours.

For example, we tend to expect that a student be mature and independent at the post-Masters level. This expectation fails often. This suggests that there should be reasonable corrective mechanisms built into curricula as well as in the organizational ethos and environment. Our M&S curriculum therefore includes a small fraction of modules on written and oral communication/presentation skills. (Incidentally, we have also experimented - sporadically and not methodically - with arranging soft-skills workshops, including unconventional themes such as time management and creativity training, with some positive effect on the students.)

A subtler issue is, however, also a better-known one: "learning to learn" versus "learning by rote". It is well-known that most seriously minded places of higher learning, need to spend significant effort to helpstudents make this transition to "learning to learn". In our context, for example, the technological component of the programme is the one that is changing at the fastest rate. This implies, among other things, that having programming skills in a given language is not enough; what matters is the ability to learn new computing paradigms and languages as and when required.

The best solution for this is to build an organizational ethos that encourages this in a genial but serious manner, and design in-class and out-of-class activities to this end. Excellent faculty and staff, together with a research-oriented environment, is one clear route to this end.

Assorted Notes

Why a Post-Masters Programme? Masters (or post-Bachelors in Engineering) is perhaps the right time for specialized training in M&S for the following reason: by then, a student has hopefully acquired suffcient knowledge from a specific knowledge domain, together with some level of mathematical, problem-solving, critical thinking, and algorithmic thinking skills, and has matured enough to understand the utility of M&S in his or her knowledge domain.

Hierarchical Content Organization: Entire content of the programme is organized on three levels of hierarchy: a *programme* of *courses* in turn comprise of *modules*. A *module* defined here as an indivisible/logical unit of content/instruction that can be meaningfully handled by a single instructor. This hierarchical organization has many advantages; specifically, it makes it possible to share modules across different programmes, allowing for sharing of instructor resources. Also, for an organization that is forced to rely heavily on visiting teachers, finding a teacher for a single module is much easier compared to finding a teacher for an entire semester-long course.

Syllabi and Student Assessment: The original conception, the syllabi included in the programme document (Arjunwadkar et al., 2007) are considered indicative of the overall scope and focus of a module, and not as rigid, sacrosanct entities that cannot be touched or altered. The actual module content can be decided by the instructor, explicitly trusting his or her expertise and judgement. In the best-case scenario, this helps alleviate the problem of dead, outdated syllabi. Similarly, evaluation and assessment have been left to the discretion of a competent teacher. With this well-intended freedom for the instructor, the entire onus then rests on finding an able person with appropriate expertise and knowledge, pedagogic skills, maturity, and academic instincts to teach a module.

The Core Courses: Survival skills in applied mathematics and statistics is quite rigorously included into the curriculum. Commonly-required areas of analysis (calculus, complex analysis, linear algebra, vector analysis, etc.), and probabilistic reasoning (probability theory, statistical inference, stochastic "simulation" methods) are the focal points of the first year of the curriculum (apart from formalisms expressly useful for modeling, such as differential equations, numerical analysis, etc.). These are handled in the first year with the dual goal of developing a strong base in modeling as well as to iron out the differences in the preparation levels of students at entry. Computing skills are predominantly geared toward developing algorithmic thinking, and large-scale code reading and writing capabilities. A mathematical formalism is expected to be presented in a three-fold fashion: domain contexts and application in which it is used, mathematical results related to the formalism with focus on concept and visualization rather than mathematical rigour, and related computational methods.

Teaching the "Art" of Mathematical Modeling: "Art" aspects, we mean, e.g., the difficult-to-teach process of arriving at mathematical models in the context of a specific problem. In our experience, the "art" aspects of mathematical modeling and the problem-centric approach are best conveyed and emphasized by exposing the students to a diverse range of problems from many domains. M&S practice ultimately rests on domain knowledge. Given the wide variety of domains in which M&S can be employed, the challenge is primarily in breadth-versus-depth trade-off optimized for the finite time available for instruction. This is best done by organizing colloquia, informal talks, and interactive case-study sessions conducted by practising experts from academics and industry alike, where the expert attempts to illustrate the advantages, techniques, and limitations of M&S in the his/her specific context.

Concept Versus Mathematical Rigour: Too much emphasis on mathematical rigour in conventional mathematics programmes, together with distancing of mathematics in typical curricula from the "reality" that mathematics can often so well describe, is believed to have harmed mathematics more than anything else: A scathing expression of this extreme view asserts that "mathematics is far too important to be left to mathematicians" (Bickley, 1964). We do not subscribe to this view; in fact, all those involved in the development of this programme have always had a deep respect for mathematics and mathematicians. However, keeping in mind that this programme is not a programme in mathematics, we find it useful to take (and propagate) the view that we are users of mathematics at varying levels of mathematical sophistication. What matters from a practitioner's point of view is the ability to grasp a mathematical result or concept in its essence, the ability to visualize mathematical constructs, and a reasonable judgement on whether one should trust one's own mathematical instincts or get help from a real expert. This implies, among other things, putting emphasis on concept instead of proof, and diving into the nitty-gritties of a proof only if it helps understand concepts better. This also puts a lot of burden on an instructor to find innovative ways of illustrating and conveying a concept. (In our limited view of literature, this outlook has been emphatically stated in Wasserman, 2004.)

Learning At a Comfortable Pace: Serious confound in traditional degree programmes, from a student's perspective, is the expectation that it is "not good" to take longer than the stipulated time period for the programme. In the rapidly changing socio-economic circumstances of this country, we are seeing students who need to support themselves financially in ever-increasing numbers. Although it is in the interest of a student to complete a degree programme in the least possible time, traditional expectations and value judgements often take a toll on a student who is serious about what (s)he wishes to learn, but cannot devote full time to education. As a standard practice at the Centre, we recommend (and help) such students to be realistic about the time they can put in, and take a judgement on how much course work they can possibly take in any semester, without putting any psychological stigma on completing the programme in more than two years. The expectation, in return, is that they excel in whatever they choose to learn.

A Wish List for the Future: We had realized very early on that a large compendium of M&S case studies would be immensely useful for this programme. In particular, each case study should begin in the problem domain, explore possible modeling alternatives and their relative merit with respect to the questions that need to be answered, whether analytical treatment of the chosen model is feasible, and if not, how does it boil down to computation/simulation. Such a "Handbook of M&S" would be a useful resource for the programme. Second, possible additions to the curriculum could usefully include a module on Fermi-like approximate and opportunistic reasoning (see, e.g., Mahajan, 2010). Third, the sector that has consistently shown the greatest amount of interest in the programme consists of working professionals who wish to further their knowledge in M&S out of necessity or interest. The logistics of making the programme available in a part-time or a web-based virtual mode is clearly enormous. This virtual mode has been on the Centre's wish list ever since the regular programme was deployed in 2008-09.

M&S in the Greater Context of Mathematics Education: Of the challenges in mathematics education has been to debunk the common myth that mathematics and formal sciences are "abstract" and "difficult", and worse still, not "applicable". We feel that this is one problem that must be addressed right from the school level. The modeling contexts associated with mathematical constructs and formalisms, and the M&S enterprise in general, is a potent candidate to dispel these myths. It might also be useful, e.g., to consider including elementary Logic at school and graduate levels. The "PQ-" logic (Hofstadter, 1979) could be a good vehicle around which concept levels can be designed for instruction at school and graduation levels. A significant barrier to mathematics could then possibly be overcome.

Acknowledgments

The M&S programme described in this paper could not have materialized if we did not have Prof. D.G. Kanhere as the founder director of the Centre for Modeling and Simulation, University of Pune. In particular, his uncanny knack to offer the right help and practical advise at the right time, his trust for a (junior) colleague's academic instincts, his wisdom in dealing with a variety of University situations, and his overall outlook on education have been an integral element in the making of this programme. We would like to thank the organizers of the 2012 National Conference on Mathematics Education (R. Ramanujam and Aaloka Kanhere in particular) for their patience and encouragement. We also take this opportunity to thank our current and former colleagues at the Centre, namely, Sukratu Barve, Prashant Gade, and Kavita Joshi, for intense discussions. Abhay Parvate, another former colleague who was deeply involved in this endeavour and contributed to the M&S programme and the Centre in innumerable ways, should have been a co-author on this paper. He, however, felt that he has not contributed to this paper enough to warrent authorship and, consequently, declined to be a co-author out of modesty and a strong sense of professional ethics.

AMV is required to include the following statement on behalf of his employer: "The opinions expressed in this article are the authors' personal opinions and views, and are in no way endorsed by Computational Research Laboratories Ltd."

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Problems of Teaching & Learning Mathematics in Bilingual Settings with Reference to Andhra Pradesh

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I discuss various problems encountered by teachers and students in the process of Teaching-Learning, the availability of study material and the problems faced by students who wish to pursue higher studies. I propose some suggestions to address the problems.

Mother tongue as the medium of instruction

I first begin with a few results and quotes from relevant researches.

1. International and local research studies in the use of languages in education are conclusive —when the mother tongue is the medium in primary instruction, learners end up being better thinkers and better learners. - RICARDO MA. DURAN NOLASCO[1], Philippines.
2. In a study, the results revealed that both public and private school children performed better both in skills and in strategies when problems were presented in their native language than when presented in English - EDUCATIONAL STUDIES IN MATHEMATICS, NETHERLANDS[2]
3. It was found that pupils and parents preferred English as the language of instruction at infant level, despite challenges faced in accessing the curriculum through the use of the second language. - GAMUCHIRAI TSITSI NDAMBA[3], Zimbabwe.
4. "English has become the medium of all relevant social interactions and the ability to use English effectively is considered an absolute essential for honourable existence." --Quotation from a retired Army Colonel, now working as a New Delhi textbook publisher[4] - Quoted in E-Journal - Teaching English as Second Language, December, 2005, Volume 9, Number 3.

Present scenario in Andhra Pradesh

The situation in Andhra Pradesh (as is probably the case in the rest of India too) is not different from the results presented above. The experience of senior teachers is that concepts are well understood when taught in the students' mother tongue. However, parents seek English medium schools owing to job opportunities that have opened up due to globalization of markets.

In the state of Andhra Pradesh, students have the choice to study in the following mediums:

<table>
<tr><th>Level of study</th><th>Medium of study</th><th>Remarks</th></tr>
<tr><td>School level
(Upto class X)</td><td rowspan="3">English or Telugu</td><td rowspan="3">The mother tongue of those who opt for Telugu medium is, generally, Telugu. Some students studying in districts bordering other states tend to study in a different language also.
Students who opt for professional courses after Intermediate (B. Tech., MBBS, Pharmacy etc.) have to study in English Medium only.
In this paper, I restrict myself to only those students who pursue Mathematics in UG, PG etc.</td></tr>
<tr><td>+2 or Intermediate</td></tr>
<tr><td>+3 i.e.
U.G. level</td></tr>
<tr><td>P. G. Courses</td><td colspan="2">Only English Medium</td></tr>
</table>

Generally, both English and Telugu mediums are offered in Urban areas while only Telugu medium is offered in rural areas. Educational institutions are managed by both Government and Private sectors. Owing to the cost of education in Private institutions, financially poor people are forced to join government managed educational institutions where the medium of instruction is, generally, only Telugu.

Advantages of teaching Mathematics in the students' mother tongue

Several researches have time and again found that the primary language at home has a strong influence on the learning of mathematics at school. In fact, the teaching-learning process is hampered if given in a foreign language because the children are forced to learn an increased number of new words in order to be able to think and express themselves entirely in the foreign language for mathematical purposes.

The meaningfulness and effectiveness of a child's mother tongue as a medium of either instruction or learning is reported in a UNESCO monograph (1953) that stated:

> *The best medium for teaching the child is in his mother tongue - psychologically, it is the system of meaningful signs that in his mind works automatically for expression and understanding - Educationally, he learns more quickly through it than an unfamiliar linguistic medium.*

Crux of the problem

In Andhra Pradesh Telugu as a medium of instruction has been introduced in Intermediate (i.e. +2 course) in the year 1969-70. A few years later it was introduced in UG courses as well. For the past 40 odd years, mathematics has been taught in both Telugu and English languages. Students who study in Telugu medium up to the undergraduate level and seek a teaching position in a Telugu medium school face no problem.

But for those students who wish to pursue mathematics as a career, the barrier of language crops in. Up to the undergraduate level, the medium of instruction is either English of Telugu. But post graduation courses are offered in English medium only.

Students who study in Telugu medium have to switch over to English medium, compulsorily, when they join Post graduation. As such some students join in English medium at 10+2 stage, while some students switch to English medium during their Degree course. And some students study their Degree course also in Telugu Medium and join in P.G. Course which has to be compulsorily studied in English Medium.

Problems in switching over from one medium (mother tongue) to another (English) include :

i) Terminology : By this time lot of terms have been learnt in one medium and it is highly difficult to follow the terms in another language. Lot of time and energy are wasted in the process. As an example several terms that appear one chapter of the Class X text book are given in Appendix I. Chapters of the class X mathematics text book are given in Appendix II. It is easy to estimate the amount of pressure one undergoes while changing the medium of instruction.

ii) Lack of adequate proficiency in English language hinders the progress in mathematics.

iii) Study material : For Telugu medium students the only study material is the text books prepared by the Government. Students are unable to refer the standard books which are published in English only.

iv) Internet facility also cannot be utilised to full extent due to language barrier.

In view of the above drawbacks, some students who are bright enough in mathematics, cannot think of joining national level institutions like N.I.T., I.I.Ts and other reputed institutions

When should the student switch over to English Medium

Translating entire research work and standard books in to Telugu language and asking the students to pursue mathematics in Telugu medium is a herculean and impossible task. (There are several languages in India)

As such the student who wishes pursue mathematics as a career, has to at some point of time switch over to English medium.

Change of medium can take place while

a) joining in Intermediate (+2) or

b) joining in Degree Course (UG) or

c) joining in P.G. course

At the Intermediate stage, students prepare for entrances of professional courses and are not mature enough to change medium and cope up with the stress. It is not advisable to change the medium while entering a PG course as the student will miss the opportunities of entering reputed national level institutions whose entrances are only conducted in English.

In view of the above discussion, I opine that the ideal time to switch over to English medium is while entering undergraduate studies after the completion of Intermediate education. An average student at this stage is 17 years old and is mature enough to adjust to the demands of the change of medium. The three year course gives a student ample time to cope with the shift in medium of instruction before entering a post graduate course. That there are no entrance exams to write or prepare for in the first two years of education gives the student some scope for stress-free learning (this is not possible, say, for a student entering Intermediate education).

Suggestions

Teaching mathematics in native language (Telugu) should be encouraged in schools. However as far as possible, all the technical terms of mathematics be transliterated. When the student is introduced to a new word, it does not matter whether it is a newly coined Telugu word or a transliterated English word. Both the words are equally familiar or equally alien. The student will have less difficulty in understanding the words, at the time of switching over to English medium.

Even for English medium students, explanation may be done in mother tongue to for better and easier understanding of concepts.

1 or 2 months course may be taught at the beginning of the course where change of medium takes place. Familiarity with technical terms, concepts, constructions of sentences in mathematics in English language,

methods of proofs in English language etc.. should be the content of the course. Suitable study material to include the above items should be prepared. One book consisting of syllabus up to class X and another book consisting of syllabus of Class XI and XII shall be prepared.

Switching over to English medium at the PG stage should be discouraged. For, the student will not have mastery over teaching in Telugu Medium nor he will become proficient enough in mathematics in English medium either to enjoy learning mathematics as a discipline or to compete with students with English medium degree.

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Appendix I

Terminology In One Chapter - Functions in Class X Mathematics Text Book in Telugu Medium.

S. No.	**Telugu Term**	**Equivalent English Term**
1	సంబంధం	Relation
2	ప్రదేశం	Domain
3	వ్యాప్తి	Range
4	పరావర్తన సంబంధం	Reflexive Relation
5	సౌష్టవ సంబంధం	Symmetric Relation
6	ప్రతిసౌష్టవ సంబంధం	Anti-symmetric Relation
7	సంక్రమణ సంబంధం	Transitive Relation
8	ప్రమేయం	Function
9	క్రమ యుగ్మం	Ordered Pair
10	సమితి	Set
11	ఉప సమితి	Subset
12	మూలకం	Element
13	తుల్య సంబంధం	Equivalence Relation
14	శూన్యేతర సమితి	Non-empty set
15	స్వతంత్ర చలరాశి	Independent Variable
16	అస్వతంత్ర చలరాశి	Dependent Variable
17	అన్వేక ప్రమేయం	One-one function
18	సంగ్రస్త ప్రమేయం	Onto function
19	ద్విగుణ ప్రమేయం	Bijective function
20	విలోమ ప్రమేయం	Inverse function
21	తత్సమ ప్రమేయం	Identity function
22	స్థిర ప్రమేయం	Constant function
23	సమాన ప్రమేయం	Equality of functions
24	సంయుక్త ప్రమేయం	Composite function
25	యొక్క శూన్య విలువలు	Zeros of function f

Appendix II

Chapters in Class X Mathematics

S. No.	Name of the Chapter in Telugu	English equivalent
1	ప్రవచనాలు & సమితులు	Statements & Sets
2	ప్రమేయాలు	Functions
3	బహుపదులు	Polynomials
4	ఏక ఘాత ప్రణాళిక	Linear Programming
5	వాస్తవ సంఖ్యలు	Real Numbers
6	శ్రేదులు	Series
7	రేఖా గణితం	Geometry
8	వైస్లెషిక రేఖా గణితం	Analytical Geometry
9	త్రికోణమితి	Trigonometry
10	సాంఖ్యక శాస్త్రం	Statistics
11	మాత్రికలు	Matrices
12	గణన	Computation

Using Mathematical Concepts for Engaging Learners in Thinking Process

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Assisting learners to get engaged in a thinking process, is the major responsibility of persons who work as facilitators at different levels, using different concepts, modes and material. The learning material or resources available for learning like text books, work books, activity and practice sheets, interactive multimedia learning objects, virtual laboratories for gaining virtual or simulated experiences, games and puzzles that are based on various concepts and process are useful for facilitators to personalize and customize process of learning. To be useful, this material should be able to invite learners to engage in critical and creative thinking process. If a facilitator takes into account process of evolution of concept and process in question, knows psychological bases of construction of conceptual understanding, familiar with the cultural background of learner and goals of education desired by society, she can design material appropriate for challenging any group of learners. This type of material helps learners to gain confidence that they are capable of,

- constructing their own understanding of concept with respect to what, why, when, etc.
- justify their reasoning,
- taking risk of explaining and accepting their own logical inconsistencies,
- sharing and evaluating their understanding by writing or narrating stories about it,
- applying and extending their understanding in various context,
- interconnecting various concepts that they are familiar with and form a coherent view of the mathematics,
- communicating their understanding with peers and others using different forms, modes, and media,
- learning from each other,
- enjoying learning alone or in collaboration with others, and
- becoming self-reliant learners.

Purpose of circulating this paper is to share the self-learning material (MLO) designed for helping learners to construct the concept "fraction" and get suggestion for enhancing its quality so that it can be used as an open resource of learning.

Introduction

While teaching mathematics at upper primary and secondary levels in schools and also working with pre-service teachers to design and develop learning activities, (work sheets, multimedia learning objects, and manipulative material) I realized that learning depends on the way activities are organized. All learners are capable of constructing their understanding of mathematical concepts if conducive learning culture is made available to them. At the same time as a teacher, I found it very difficult to manage time and energy for engaging learners, in face to face mode situation, in intellectually demanding activities. To manage this problem I started developing learning material (learning through reading available material, experimenting, and discussing) for pupils of primary and upper primary classes. This material is designed and developed using my personal experiences related to my own understanding, my problems of learning as an average learner, my school teaching-learning experiences and researches related to learning of concepts under consideration.

Using experimental design I studied (1994-95) impact of this kind of learning material developed (dialogue between students and teachers) on the concepts circle, measurement of segment, concept and measurement of perimeter and area, measurement of angle, algebraic expression and operations on algebraic expressions. Schools selected for this purpose were those educating children belonging to "meager and irregular income" group. Pupils were asked to read the material during supervised study sessions in pairs or triads, do activities and discuss them in small groups and sessions were ended with large group discussions. Pupils were supplied with the material required for experimenting to answer questions that teacher asked or for doing related class work. Students were found to enjoy these self-learning activities and as a teacher I got ample time to develop rapport with all learners. This provided me an opportunity to study their meaning making processes and its relation with the language (sentence structure, terminologies used in the material) of the material, illustrations used, form and the stance taken by me as facilitator in absence. Sixty five percent pupils showed qualitative development (know what, why and how with respect to concerned concept and procedures) in their performance. The classes, in which teachers used this material with enthusiasm, students were found to develop interest in learning all subjects. Few teachers realized the importance of this kind of material for engaging learners, especially when they have to engage class but also required to complete other non-teaching responsibilities. Many teachers reported that they found the material useful but time consuming, as pupils were working at their own pace. For many others it was a waste of time and energy as in examination no one asks questions related to understanding of procedures.

Same material along with material developed on other mathematical concepts was used for assisting pre-service teachers to design and develop electronic multimedia activities for practicing teaching in regular classes. Pre-service teachers were asked to read teacher-student dialogue and discuss activities incorporated with it, in small groups along with various aspects of teaching-learning process. This helped pre-service teachers to relate theories of learning with learning material and put them into practice. Using experimental design I studied (1996-98) impact of this material on pre-service teachers' learning and teaching performance using qualitative methods. Student teachers found this material useful for understanding concept of learning. It also helped to them to enhance their understanding of mathematical concepts and improved their own process of learning.

As a result of working with school learners, pre-service and in-service teachers for more than thirty years, I realized that teachers and learners share similar understanding (misunderstanding) or rather information with respect to different mathematical procedures. Though teachers talk about concept, many of them fail to understand concept of "mathematical concepts". Teachers easily get confused when they are requested to explain procedures that they are using. Teachers while teaching, concentrate their efforts on making learning (content) very easy for learners as they underestimate capacities of learners. For most of the teachers, "repeating information" orally is same as "explanation". This easy learning is based on remembering dos and donts type rules (procedural information) to be followed while working out procedures for solving exercises given in the text books, without asking questions or raising doubts. Few learners and teachers are interested in asking questions like, how, why and when these rules came into existence, who made these rules, why one felt necessity of making these rules, why are they important for those who are not interested in learning mathematics etc. It is also important to note that many of these rote learners having mastery on procedures, become graduates and masters in mathematics.

If I work with a child learning in fourth class and discuss any mathematical concept with her, she explains me exactly like her teacher. She gives me some examples to make me understand (follow rules) that are given in her textbook and that are supported by her teacher. If I start experimenting with figures, concrete objects or numbers to show her my understanding of the point being discussed, she orders, "Don't waste your time in doing this. Write etc., etc., and finish the work… that will save your time." If I ask some questions related to certain aspects of the concept that is being discussed, she requests, "Please don't make me confused by asking such questions. Do as I tell you… if you want to understand and get correct answers." She also instructs me to remember the steps of the processes involved in the completion of stereotype exercises. With this kind of learning experiences she tops as a best performer in her school.

Same attitude prevails in teacher education classes. Pre-service teachers and teacher educators find it difficult to realize importance of developing thinking ability through formal learning activities. Each year I had to conduct many sessions to assist them to understand importance of acquiring thinking skills and strength of mathematical concepts as a tool for acquiring and mastering various thinking skills.

Similar environment exists in education departments of corporates (or CSR) that are involved in producing self-learning multimedia material for school learners. Here teachers, graphic designers and instruction designers dominate the decision making process who have their own beliefs about making learning easy and interesting. It is also observed that to make the product salable, production houses are forced to design material that repeats and replicates the content (text) given in the text book. They are expected to add attractive animated graphics to it. Many times these graphics are mathematically confusing or conceptually incorrect. Each concept is dealt as if it has nothing to do with other related concept. Teachers who select the material for learners compare the text, graphics and voice over of multimedia objects on the basis of the text books prescribed by the institution.

I had designed and developed some learning objects for one institution. In some of the multimedia learning objects (MLO) character of a "learner" was shown as a meaning maker, thinking loudly and explaining new learning to her. I did organize a demonstration lesson for teachers of the concerned institution using this type of MLO. In this MLO meaning maker reads the statement or definition from the text book, raises some questions, starts analyzing the text on the basis of previous understanding, strikes out irrelevant words, highlights important unfamiliar words or attributes of the concept, organizes information with reasoning, experiments with related concepts with which she is already familiar and with those she thinks have direct or indirect connection with. Along with this she gives justification of her choice, reorganizes information in the context of concepts, finalizes the concept map of her understanding, discusses how the concept can be used and finally leaves the screen by reviewing whole process. Students were requested to identify themselves with the "meaning maker" and think along with her and do activities using their own understanding of concept with which she is working.

After the session was over students were asked to raise questions but no one said anything and session was over. The expert commented that it was very dull sessions as students did not ask any question to the teacher. For her a good teaching involved lot of telling by teacher and listening and questioning by learners. But at the end of this three day workshop, many students opined that they liked MLO and enjoyed to be with the MLO character very much and it is the most useful learning. It helped them to realize importance of learning by reading text-book. After demonstration session was over they tried to use text book as a self-learning material and enjoyed individual as well as group learning.

Many persons engaged in various activities related to teaching-learning process feel that learning should be restricted to (collecting) rules for conducting different procedures and remembering them is easy and shortest way of learning. It is sufficient for passing examination with good scores. This helps any average person to score good marks in mathematics without understanding concepts. Getting involved in asking questions like why, what, when and so on is like inviting unnecessary trouble. This rule learning approach is helpful to learners who understand them superficially and remember them without much struggle. They even pass higher level mathematics courses without any problem. But many learners find it difficult to remember these rules without basic or fundamental understanding. They develop fear with respect to learning mathematical concepts and processes. Thus teaching of mathematics becomes an easy job for any person, who helps learners to practice mathematical procedure and remember them till examination.

This situation necessitates the designing of learning material that may be of some help to any learner to understand relevance of learning mathematics (set of interrelated concepts) in ones day to day life as well as a process to be done for acquiring and practicing various thinking skills. This paper discusses how concept "fraction" can be used as a tool for assisting learners to acquire various skills.

Concept "fraction" as a tool for developing various skills:

Younger children have a capacity to compare the objects on the bases of various quantities. This capacity is enhanced with the command on number system. With the help of numbers one can assign values to the properties of objects and this provides accurate evidence for justifying comparison. Measuring instruments make use of number system and provide valid and reliable values for making inferences. Fraction as a mathematical concept provides us a valid and reliable system for mathematically equal distribution and comparison of quantities for different purpose. All people engaged in different professions use fractions for different purpose and using their own ways of fractioning wholes. People who did not attend school also have their own methods of handling fractions. At primary and upper primary level school level, concept "fraction" offers many opportunities for engaging oneself in meaning making and decision making process. For making meanings, making decision, and communicating ones understanding one needs to get engaged into deep learning.

Fraction as a mathematical concept

"Fraction" as a concept based on some basic assumptions like all other concepts. These assumptions are not discussed or emphasized while discussing concepts in formal classes. Adults and teachers using fractions apply these assumptions unconsciously but are not explicitly aware of them. For example, to think about fractions we have to assume "a whole" to begin with. This "whole" assumes different meaning in different contexts. Unless we assume or define what is "whole" mathematically, there is no scope for defining a "fraction". Once the "whole" is fixed or defined, it is possible to define "fraction" with respect to it. For example, if two equal objects are assigned value one, then with respect to this value, one of the objects can be assigned value "half" and is represented as ½. This representation ½ is a rational number.

Thus person who needs to use fraction decides and defines the "whole and part" as per the demand of the context. This aspect of concept "fraction" rarely becomes a part of classroom discourse as it is not discussed in the text books. There are many such aspects of the concept "fraction" that are not discussed in the textbook or during the class discourses. But for gaining deeper insight there is need to emphasize these aspects during classroom discourse. This assumption gives rise to many questions like, if one wants to use fraction ½ as an operator that is multiplier of other fraction then is it necessary to have both fractions of equal or similar whole, or can we have these "fractions" from different "wholes" will arise. Experimenting with fractions to get answers to these types of questions will assist learner to get deeper sense of number various systems. Students enjoy debates on this type of questions.

Various aspects of concept "fraction"

Concept- Fraction	Skills involved: decision making, meaning making and self-explaining
Whole-part relationship	Whole division of whole, equal parts or shares representation of value of parts using numbers
Relative nature of concept "fraction"	application of mathematical idea of fraction in day to day life- uses and limitations
Representing fraction using whole numbers and rational numbers	meanings that can be assigned to fractions as a number fraction made up of part having equal value
Discrete quantities	use of whole-part relation

Continuous quantities	use of whole part relation
Decimal fractions	dividing whole into hundred equal parts
Equivalent fractions	use of terms that describe relationship between fractions properties of equivalent fractions
Like fractions Unlike fractions	use of terms that describe relationship between fractions
Comparison of fractions	process of comparing fractions, all value should be considered with respect equal wholes
Mathematical operations	processes for conducting mathematical operations developing self-explanation
Ratio: proportion	comparison of two interdependent and similar quantities and use
Ratio: variation/ rate	comparison of two interdependent and different quantities and use

Designing of self-learning material for developing understanding of "fractions"

While designing material, efforts are made to invite and assist learners in constructing their own meaning of "fractions". Pupils are not expected to attain any particular concept or acquire any particular skill at one learning instance or at the end of any discrete learning cycle. In other sense objectives of MLO were not clear as per behaviorist tradition. Opportunities are provided for pupils to test their understanding and feedback is provided indirectly through some other part of the learning material. Thinking process is emphasized while designing and developing stories that uncover fundamental aspects of concept fraction. MLO aims at assisting learners to revisit different concepts that they are familiar with and use them for gaining new or deeper understanding. They are expected to be with characters who are engaged in meaning making and explaining their understanding to self.

A sample of self-learning material (MLO- content) is given below.

Fraction – part 4

What do we mean by a process division? What do we mean by equal parts? Are we not familiar with the process of equal partitioning?

Yes! We can cut or divide whole into equal parts.

What can we do if we are asked to distribute twenty bananas equally among five families?

During each round we will be giving one banana to each family.

For the first round of distribution we will need five bananas. Likewise we will need five bananas for each round of distribution. Thus twenty bananas will get distributed in four rounds. No banana will be left.

Divisor = 5 → number of groups that should have equal number of bananas

Dividend = 20 → number of bananas that are available for distribution

Quotient = 4 → number of bananas in each group

To begin with we had 20 whole bananas. We considered that all bananas equal. We distributed these bananas in five equal groups. Now there are four bananas in each group. Not a single banana is left now. The group that is formed after this division is also a "whole".

What will happen if we are asked to distribute 21 bananas equally among five families? Will each family get all whole bananas? If all families discus together and decide to give 21th banana…whole banana to Jaya who is very weak, then will that be an equal distribution? Jaya is a member of one of the families. Will it be an equal distribution if 21th banana is cut into five equal pieces and one piece is given to each family?

Discuss these questions with your friends and write your thoughts in detail.

Here is a papaya weighing 1 Kg. Here is mother and her three month/ one year old child. How will you distribute it equally to mother and her child?

Write your answer in detail.

Oho! Look here. Baba, Aai and a child are having tea. They are having three equal portions of a cake. To share it equally, each one should take one piece. But child likes cake very much and wants to have one more portion. Mother gives her portion to the child. Now child is feeling uneasy. Why has this happened?

Look here, Mr. Gundeshwar, the minister for food supply and distribution, and his secretory Mr. Babu, are engaged in a serious discussion. They are deciding about the quota of ration to be given to each person. Let us listen to their problem.

Mr. Gundeshwar: *Mr. Babu, a person above 12 years will get one unit of ration and a child below 12 will get half unit.*

Mr. Babu: *How will half unit of ration be sufficient for a child below 12 years?*

Mr. Gundeshwar: *Why…? Don't they charge only half ticket for children…?*

Mr. Babu: *Sir, children require more food as they grow physically. They play and exercise constantly. They require more food. In bus they require less space therefore they need to pay half charge only.*

What is your opinion on this issue? Collect needed information and write a letter to Mr. Mantreeji that explains your *stand on the issue.*

Now let us review our understanding of concept. We decide what is "whole". For example we take a piece of paper (area) and call it a "whole" (area) or what we call it as "one" (area). We take a bunch of some bananas and call that bunch a "whole".

When we divide whole mathematically, we mean that we are making equal parts of the whole under consideration. Whole is usually identified by value "one".

Part is described with respect to "whole". It is described using a "fraction", a number. Fraction is represented using number to denote equal parts that are cut from a "whole". It is called a denominator. Numerator of fraction indicates parts that are being considered. The line that separates numerator and denominator indicates division processes. For example, ½ is a fraction that says that whole is divided into two equal parts and only one part is considered. Remember though the "whole" means "one", and is divided into equal parts it is not indicated while writing fraction.

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Poster Presentations

A Dialogue on the Effective Interplay of Skill and Knowledge in Math- Education at Undergraduate Level Revisited

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Abstract: The genesis of the original paper, which was presented at National Conference held in Banaras Hindu University on Mathematical Modeling and Computer Simulation, March 25-27, 2011, goes back to the 1990's, the decade just before the onset of new millennium. It was observed, that demand for knowledge based education has shifted to skill based education in India. Many Indian Universities decided to go with the demand and initiated undergraduate programs which were a mix of many subjects, but did not provide concentrated learning in a particular direction. However, is it possible to sustain growth with skill alone without knowledge base?

In this atmosphere, Mathematics, which is considered by general public as a knowledge based study in spite of the fact that every other technology today is 'digital', took a back seat. We, the Mathematicians in India, failed to design an undergraduate course which provides depth of knowledge as well as skill to apply the knowledge. Knowledge is required for future growth and skill is necessary to sustain oneself under the demands of the ever changing world.

In the present paper while I am glancing through the same premise, the solution suggested is based on a broader outlook. Also I intend to take up a pilot survey in certain Mumbai colleges based on the research survey done in the paper titled "Applicable Mathematics in Mathematics education" a Viennese research project by Hans Hamenberger (Austria) & Dortmund (Germany), and present the conclusion obtained through the survey.

The point is that the under graduate education in Math should be so designed that it exposes the collaborative nature of Mathematics with technological, commercial and research work. Acquisition of Math education helps in good governance as far as today's society is concerned.

The Need to Reform the Undergraduate Mathematics Program

The main contention of Mathematics educator in India today is that the students who take up Math as an optional subject of study are hardly inclined towards the subject.

Everyone can learn Mathematics provided s/he has an inclination to learn. By the time the student has finished high school/ twelfth standard their interest and abilities are already directed. But very few students who really can handle Mathematics with ease take up undergraduate study in Mathematics. The deficiency is caused by the fact that we have failed to attract mathematically inclined students towards Math graduation. The reason for such a situation is apparent in the remarks that the youngsters make "the only prospect that a math graduate/postgraduate has is in the teaching line." The abstract nature of undergraduate syllabus adds more fuel to this idea. And therefore students with numerical ability move on to study different subjects like chemistry, economics, engineering etc., hoping to get more lucrative career.

The other major hassle for a math teacher is a huge class strenghth of say hundred students with only a few mathematically inclined students. Ninety percent of these students fail miserably. In such a situation the subject matter gets diluted, in an effort to promote the majority of the students to higher class.

Suggested Reforms

Reforms could be brought about in two folds

1. Attract mathematically inclined students to take up mathematics.
2. Better communication of mathematics.

We need to attract mathematically inclined students by showing better future prospects for a Math graduates. If we can give the students what they desire half the battle for pedagogy at undergraduate level is won (Rogers, 1969).

When a student takes up Mathematics as a subject of study which requires continuous endeavour and concentrated hard work to progress, s/he must have a very wide range of job prospects.

This is possible only if there is an effective interplay of skill and knowledge in the prescribed syllabus. Introduction of Applicable Mathematics at every stage of Math learning at undergraduate level would bring about the change. "We could say that it is not so important that students learn applied Mathematics, but they should learn how to apply it".Hans Freudenthal(1973).

In the current scenario syllabus of undergraduate study, students are exposed to various branches of Math which are applicable , but they hardly learn to apply it.

Suggestion therefore is that during the progression from fist year level to third year level, a learner could be exposed to applicability of Mathematics in a Math-Lab by working on projects and case studies etc which utilises mathematical concepts.

At third year level the learner may take up a "co-op" project to use a specific math topic, with an industry, for which student earns credits. Cooperative education is a structured method of combining classroom-based education with practical work experience[1]. This system is adopted by several universities in the USA. A co-operative education experience, commonly known as a "co-op", provides academic credit for structured job experience. This system of education will give the students hands on industrial knowledge/experience and would enable smooth transition from college to work.

Following is an excerpt from a Canada based university website[2]. There are 3 partners in Co-operative Education, all three benefit from the partnership:

1. The student gains valuable work experience, earns a salary that can assist in financing his or her education, is exposed to different jobs in varied locations and can integrate academic training with practical experience in the same field.
2. The educational institution, by liaising with industry, business and government communities, is assured that its curriculum is relevant and that its graduates achieve a high level of focused academic and professional competence.
3. The employer gains a highly motivated worker with good general skills and gets the chance to evaluate students for potential permanent employment. In addition, the employer can become a partner to ensure the education system is responsive to changes in the economy and market.[4]

This system could be modified to suit Indian scenario.

The graduating student could be given honours degree if s/he has earned extra credits by completing a co-op project, based on her study material after completion of third year degree program. In the mean time those students who want to pursue academics and research in mathematics may go ahead with class room study of deeper abstract concepts and take up a postgraduate program of two years.

Improvement in Math Communication

The first and foremost point in this connection is that a Math class strength should not exceed 50. Introduction of applicable math is possible only if the style of pedagogy is "student work teachers facilitate". With a large group of students in a class it is not possible. Today we do produce a large number of graduates with math as their major option, but very few of them know how to use it, which is a huge waste of resource.

The Mathematics educators could be trained to handle applicability of Mathematics.

Discussion

I wish to keep the paper open for discussion, after presenting results of a pilot survey which I am in the process of conducting, in the lines of above discussion.

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Math Art and Science: An Experiment in Mathematics Laboratory Activities with Tangram

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Introduction

The laboratory work in mathematics can provide linkage between mathematics, which is regarded as an exact science and Chinese art of Tangram. The seven pieces or tans can be used to create many recognizable shapes by flipping, turning, orienting and locating the pieces. An attempt was made to provide systematic laboratory experiences to the students at upper-primary school level by using the activity sheets formatted into the reporting style of practical work. Not only the intervention solved the problem of organizing mathematics laboratory work but also provided the students of an experience of linkage of mathematics to art work and culture of another country.

The way mathematics is generally taught makes it a dry, drab and abstract subject. Often is felt that for learning mathematics with understanding, it is essential to make students have hands-on experiences providing a feel for the processes of observation, transformation, proof, analytical thinking and communication. Especially, in geometry, there is a dire need to exposure children to activities and exercises with shapes that provide students the opportunities to observe, visualize, draw, measure and compare shapes in various positions. Such activities can be helpful in sharpening students' spatial skills and perceptual learning of geometry.

Many educators hold that working on shape puzzles can be mentally stimulating and enjoyable. It is believed that mathematical puzzles are as old as mathematics itself (Champanerkar1, 2004).But, puzzles tend to remain as leisure time pleasure activities for a select few only. The puzzles with shapes of triangles, squares and parallelograms have facilitated the interested young and grown-ups alike since they make interesting patterns and new shapes when combined, slid, turned and flipped.

Servais and Vagra (1971) say that most students leave school without having felt the beauty of mathematics. They advocated that teaching reforms and innovative classroom practices are required that should help students enjoy mathematics by providing a freedom of expression to them which arises from playful activities. According to them, "Mathematics recreations also have a role in helping pupils to like mathematics. Puzzles in mathematics are similar to songs in music: short and self –contained… They are also like anecdotes, which often point to deeper ideas… Puzzles can be excellent points and incentives for deep ideas in school mathematics."

What is required is that shape puzzles which are the part of some popular books need to be made the tools to learn formal mathematics to combine pleasure and studies. Since shape puzzles can bring art and design in mathematics activities and thus make mathematics an 'Art', and inclusion of the element of 'finding out' in such activities in a laboratory setting can make it a 'Science'. The present innovative initiative and educational experiment stems from a requirement of presenting mathematics as art and science both using tangrams.

Rationale for the Intervention

The rationale for the initiative 'Math Art and Science' at upper-primary level is to make learning of spatial aspects of geometry interesting and investigative. To make learning of geometry meaningful it is essential to link it to other discipline like processes of science, art and multicultural awareness - Chinese culture in this case.

Creating tangram figures and designs has the potential of providing opportunities for combining art and science along with perceptual learning of geometry. Keeping this in view, it was thought to create to mathematics laboratory activity sheets and make tangrams a pedagogical tool.

Development of Laboratory Activities Manual

For developing the laboratory activities, the available puzzles books which had assorted collection of puzzles were scanned. These books where in the popular format having a collection of puzzles, tricks, teasers and mindbenders.

The educational challenge here was to identify the relevant material and convert it into activity sheets for laboratory and art work in a formal school setting.

The kinds of tangram that were available were more for creating artistic figures and designs.

Nonetheless, when tagrams done as systematic activities, they had the potential of combining aesthetics and spatial skills of sliding, turning and flipping of shapes by creating artistic figures. The creation of geometrical shapes from the tangram pieces and making observations, measurements and drawings were undertaken.

An analysis of the available literature, plausible activities and available school time (25 lesson sessions), the following sequence to the laboratory course was followed for transacting the present laboratory course

1. Demonstration of making a tangram and introduction to the laboratory course by two exemplar activities by the teacher
2. Development of spatial and artistic sensibilities by creating human figure with the tangram pieces
3. Measuring lengths, areas and perimeters of the tans and comparing them to the global square of the tangram
4. Creating geometrical shapes with select tans and all the seven tans
5. Activities for convex polygons from the tans

Deciding about this sequence was important from the point of view of principles of learning, once, the sequence was decided, the structure of each activity as thought about and it was kept simple to suit the children at upper-primary level which include: (a) Purpose (b) Material Resources; (c) Procedure; (d) Observation; and (d) Result.

The list of the laboratory activities was as follows.

1. To make the given figure of a kicking man with the help of seven tans.
2. To make the given figure of a running man with the help of seven tans.
3. To make the given figure of a stepping man with the help of seven tans.
4. To make the given figure of a standing man with the help of seven tans.
5. To make the given figure of a beggar man with the help of seven tans.
6. To make the given figure of a knealing lady with the help of seven tans.
7. To make a square with any two pieces of the tangram.
8. To make a square with any three pieces of the tangram.
9. To make a square with any four pieces of the tangram.
10. To make a trapezoid with any four pieces of the tangram.
11. To make a square with any five pieces of the tangram.
12. To make a square with any six pieces of the tangram.
13. To make a square with all the seven pieces of the tangram.
14. To make a trapezoid with all the seven pieces of the tangram.
15. To make a rectangle with all the seven pieces of the tangram.
16. To make a parallelogram with all the seven pieces of the tangram.
17. To make a triangle with all the seven pieces of the tangram.
18. To make a large triangle with any three pieces of the tangram.
19. To investigate the relationship between the small triangle and global square of the tangram.
20. To investigate the relationship between the medium triangle and global square of the tangram.
21. To investigate the relationship between the small square and global square of the tangram.
22. To investigate the relationship between the parallelogram and global square of the tangram.
23. To investigate the relationship between the larger square and global square of the tangram.
24. To make two convex polygons with two small and two medium triangles and find the ratio of their perimeters with that of the global square.
25. To make four convex polygons with all the seven tans and find the ratio of their perimeters with that of the global square of the tangram.
26. To make twelve polygons with all the seven tans and find the ratio of their perimeters with that of the global square of the tangram.
27. To create and display any three art pieces with tangrams.

Thereafter, a student's laboratory manual titled 'math art and science' was made. The manual had instructions in laboratory work format to carry out the activities. Keeping the cost factor in view, the manual was handwritten, photocopies was distributed to the students. The materials for doing the laboratory activities consisted of figure outline cards, coloured card sheets, scissors, pencil, scale and glaze paper.

Implementation

The project was implemented as weekly laboratory period in the timetable. The thirty five students of grade eight participated in the project for one complete session. It was initiated with a demonstration by the teacher in which general information about the Chinese puzzle of tangram, drawing a tangram and cutting out the tans.

Also, a cat using seven tans was made by the designer of the project using all the seven tans during the demonstration session. The terms like sliding, turning and flipping of shapes were introduced and explained.

The students were told they will be evaluated by giving grades A, B and C for their laboratory work. After this, the laboratory work was done by the students weekly basis throughout the academic year.

For making tans, the following instructions were given: (i) Take a piece of coloured card and draw a ten-centimeter square on it. (ii) Draw the pattern for the tans and cut along the lines to get seven pieces

The students kept a record of their laboratory work which was signed by the teacher. The selection of three art pieces for the last exercise was to be done from the given chart made from the internet images.

Outcomes

The development of the material that combined the art and science of mathematics in the form of students' laboratory manual was an important outcome of the project. It sharpened the skills of the implementer in accessing information, adapting it and using it for introducing the home grown laboratory work. One of the important outcomes of the project was the hand written activity sheets prepared for the future use.

The students reactions (N=35) were found to be significantly positive on their favorableness for the project on three point scale of favourable, indifferent and unfavourable. The chi-squared test was used for the analysis.

Also, students typical comments were collected on the project where were as follows :

- *I have learnt the steps of experiments in mathematics through this intervention.*
- *This practical work helped me to learn the spatial aspects of mathematics and art work.*
- *The Chinese art with tan-pieces was the high point of this exercise.*
- *My interest in the Chinese art forms increased and I learned more about tangrams from the internet.*
- *To me it appeared that there is both science and art in every branch of knowledge and both must go together.*
- *Mathematics perhaps is a basis human sense for the abstract and can be made appealing by linking to other aspects of life.*

Thus, the laboratory work can easily be carried with simple pieces of apparatus like tans for providing the science–related experience in mathematics. For the laboratory work, it is the processes of finding out and verification that are important rather than the use of market-driven equipment and devices.

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Teaching the Concept of Limits

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In the NCERT Math textbook for Class 11, the concept of limit is first introduced in the context of derivatives. But actually the idea of limit is very general and differentiation is only one specific instance where the notion of limits is essentially used. We believe that a good understanding of limiting processes, built up through various examples is a prerequisite for a course in calculus. With this in mind, at various places in the high school math texts of Kerala State syllabus, the basic features of limiting operations are introduced in an intuitive manner without formally introducing the concept of limit.

For example, decimals are first introduced in Class 6 as an alternate way of writing fractions with a power of 10 as denominator, such as writing 1.23 for $\frac{123}{100}$. In Class 9, this is extended to a sequence of fractions with denominators powers of 10 which approximate a given fraction better and better, such as writing 0.333 ... for $\frac{1}{3}$, since the sequence $\frac{3}{10}$, $\frac{33}{100}$, $\frac{333}{1000}$, ... gives better and better approximations. Irrationals are introduced in Class 9 by first noting that the square of the length of the diagonal of a square of side 1 unit is 2 and then discussing Hippasus's proof that there is no rational number whose square is 2. This leads to the introduction of a new number $\sqrt{2}$ to denote the length of this diagonal. It is then noted that the squares of the numbers 1; 1.4; 1.41; 1.414; get closer and closer to $\sqrt{2}$. The notation $\sqrt{2}$ = 1.4142 ... is then introduced as an abbreviation of this fact.

In the chapter on circular measurements in Class 9, the problem of computing the perimeter of a circle is introduced by first observing that a circle is approximated better and better by inscribed regular polygons of more and more sides and then proving that the perimeters of such polygons are proportional to the diamter of the circles. This leads to the conclusion that perimeter of circles are also proportional to the diameters and to the introduction of the number as the constant of this propotionality. The sequence 3; 3.1; 3.14; . . . is then given as approximating this number with increasing accuracy.

Madhavan's squences for approximating is also discussed as an aside.

In Class 10 the idea of a tangent to a circle is introduced as limiting position of lines drawn through a fixed point outside the circle. While analysing these cases we can observe that limit is not only a quantitative change,but it is qualitative like limit of a sequence of rational numbers become an irrational number. This is why limit is used as a tool in various situations. We believe that the consolidation of these ideas and their extension to limits of functions leads to a more effective teaching of calculus.

A Study on Attitude and Achievement of Higher Secondary Students in Mathematics in Relation to their Gender and Stream

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Introduction

Mathematics

The ATOM has been conquered and outer pace penetrated. It is mathematics that has pushed and is pushing forward the frontiers of scientific and technical knowledge, discovery and invention.

Mathematics now dominates almost every field of one's activities. In this age of science and technology, it has permeated through the human life in such a way that, it has now become every man's everyday concern. Mathematics disciplines the mind, systematizes one's thought and reasoning. The subject has also rich potentialities of affording true enjoyment to its students.

Throughout the centuries, mathematics has been recognised as one of the central strands of human intellectual activity. From the very beginning, mathematics has been a living and growing intellectual pursuit. It has its roots in everyday activities and forms the basic structure of the highly advanced technological developments. It comprises intricate and delicate structures which have a strong aesthetic appeal. It offers opportunities for opening the mind to new lines of creative ideas and channelling thought.

Significance of Mathematics

Mathematics is an important subject in school curriculum. It is more closely related to one's daily life as compared to other subjects. Except one's mother tongue there is no other subject which is more closely related to one's daily life as mathematics. Mathematics is considered to be the queen of all sciences. Napoleon remarked that- "The progress and improvement of mathematics is linked to the prosperity of the state".

In this context Kothari Commission (1964-66) suggested that "Science and mathematics should be taught on a compulsory basis to all pupils as a part of general education during first ten years of schooling."

Value of Teaching the Subject Mathematics

Values are regarded as desirable, important and are held in high esteem by the people who live in a particular society. Thus values give meaning and strength to a person's character by occupying a central place in his life. Values are the guiding principles of life which are conductive to all round development. Therefore values reflect one's personal attitudes, judgments, decisions, choices, behaviour, relationship, dreams and vision. Mathematics helps in attaining and developing various values amongst the children.

Attitude

Attitude in the narrow and more specific sense are essentially motor sets of the organism toward some specific or general stimulus. Although there is no standard definition of the term attitude, in general it refers to a learned predisposition or tendency on the part of an individual to respond positively or negatively to some object, situation, concept or another person.

In assessing Mathematics performance and potential of students, attitudes towards Mathematics and Mathematics learning are frequently cited as factors contributing to success. However, an individual's attitude towards Mathematics can be influenced by many factors. Two such factors which have been the focus of investigation are the individual's sex and socio-economic standing. It is generally held that females exhibit less positive attitudes towards mathematics than males do.

Objectives of the Study

The following objectives have been formulated related to the study:

1. To study gender-wise difference in different components of a student's attitude towards mathematics.
2. To study stream-wise difference in different components of a student's attitude towards mathematics.
3. To find out the relationship between attitude of a student towards mathematics and his/her academic performance.

Hypotheses of the Study

Keeping in view the above objectives of the study, the following hypotheses have been framed.

1. There is significant difference in different components of attitude of students on the basis of their gender.
2. There is significant difference in different components of attitude of students on the basis of their stream.
3. There always exists a positive relation between achievers in Mathematics and their attitude towards the subject.

Methodology, Sample, Tools

To study the present problems following methods have been adopted: 1. Attitude Scale. 2. Personal Interview. For the present study 100 students (which is one third of the total population) have been selected randomly from the provintialised colleges of Guwahati, those studying at Higher Secondary first year (10+1 level). In this study following tool have been used to collect data:

"Attitude Towards Mathematics Scale" (ATMS), developed by Dr. S. C. Gakhar and Rajni, Department of Education, Panjab University, Chandigarh in 2004.

Review of Literature

D. Stipek and H. Granlinski (1991) indicates in the article that girls have lower expectations for themselves in math than boys, and that girls believe they do not have mathematical ability. When girls do poorly in math, they attribute their poor performance to their inability to do math. This study explores the beliefs of third-graders and junior high school students (male and female). It shows that girls' beliefs begin early in their education and persist into junior high school (and probably beyond). Therefore, starting at the elementary school level, teachers need to i) encourage girls to have higher expectations for themselves in math, and ii) offer girls alternative, positive explanations of their math performance.

B. Moore (1993) found in school students that impression of the teachers as 'like' or 'smart' significantly predicted student's attitude towards Science and Mathematics. The results of the study also showed that 'boys' are more advantaged than their counterpart 'girls'.

J. Gill (1994) indicates that middle school and high school girls have positive attitudes toward school but negative attitudes toward mathematics. It focuses on the gendering - the separation of boys and girls - of Australian schools through the study of 7th, 8th, and 10th graders in coeducational programs as well as girls-only schools. With regard to teachers, the paper suggests that separating boys and girls during math instruction does not improve girls' negative attitudes toward math.

E. Fennema and J. Sherman (1995) found that students of teachers who were organized, achievement oriented and enthusiastic tended to have more positive attitude towards mathematics and science.

D. Swetman (1995) shows that girls' positive attitudes towards mathematics decline as they grow older. Initially girls have more positive attitudes towards math than boys do, but as they continue in school, girls' attitudes become more negative. In order to improve girls' performance in math, teachers need to facilitate positive attitudes in girls towards mathematics.

X. Ma and J. Xu (2004) conducted a study to determine the casual ordering between attitude towards mathematics and achievement in mathematics of secondary school students. Results showed the achievement demonstrated casual predominance over attitude across the entire secondary school. Gender difference in this casual relationship was not found but elite status in mathematics moderated this casual relationship.

S. Saha (2007) conducted a study on gender, attitude to mathematics, cognitive style and achievement in mathematics. It was found that all the three contributes to statistically significant difference in achievement in mathematics.

S. Manhas and M. Sharma (2008) studied on the attitude of the students as a correlate of their academic performance. They found boys hold more positive attitude than girls towards mathematics. They also found significant difference existed between male and female with regard to their personal confidence, perception of teacher's attitude towards mathematics. More males than females perceive mathematics to be a male dominated subject.

R. Ravanan, A Blessing Mary and Julie have studied on attitude towards mathematics of XI standard students in Trichy. They considered 450 students of XI standard from 10 schools in Trichy district (Four government schools, three aided and three unaided schools from both rural and urban areas. The Major Findings of the study are:

There is no significant difference in Attitude towards Mathematics of XI standard students in Trichy district owing to differences in their Gender, Region and Medium of instruction.

There is significant difference in Attitude towards Mathematics of XI standard students in Trichy district owing to differences in their Stream of study, Types of school Management and Socio-Economic status.

Analysis of Data

Attitude of students (studying at 10 + l level) towards the subject mathematics have been collected with the help of questionnaires have been scored adopting the procedure given in the manual. The analysis has been based on eight components: Wider applicability, Development of skills, Reasoning, Objectivity, Intellectual development, Individual outlook and Universal outlook. For the purpose of analysis marks scored in mathematics at High School Leaving Certificate Examination (H.S.L.C.) have been taken as student achievement. For interpreting and analysing the collected data basic statistical tools and SPSS have been used.

Major Findings

1. Male students show more positive attitude towards mathematics than female students.
2. Students' attitude depends on stream.
3. There is a very low correlation between attitude and achievement of a student.

Conclusion

As male students show higher positive attitude than females, importance should be given towards the study of mathematics for female students. It should not be considered mathematics as a subject for boys. Girls should encouraged to study mathematics. Present study demands reform in the present examination system as it reveals low relationship between attitude and achievement.

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A Study of Mathematics Anxiety viz-a-viz Motivation, Performance Approach Orientation, Problem Solving and Impact of Pedagogical Intervention Among Pre-service Teachers of Delhi

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Introduction

Schools as powerful agents of formal education should be structured and tailored as per the demands of buddy generation. Academic improvement is the unavoidable aspect of education the focus therefore needs to be to alter the misconceptions among learners. It is very clear that the attainment of mathematical ability is expected out of basic education and literacy. The core areas of learning are otherwise called as 3Rs; Reading wRiting and aRithmetic. In order to ensure the basic minimum, teachers should have an attitude to do and understand the basics of mathematics.

The National Council of Teachers of Mathematics (NCTM, 2000) (in US) posits that the role of conceptual understanding in learning and building mathematical proficiency has been well documented. "Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge" (NCTM, p. 20) Students need to learn Mathematics with understanding and not only for gaining procedural fluency, Mathematics equips the child with certain powerful set of tools to understand and change the world (Haylock and Thangata, 2007). Mathematics teaching develops certain form of critical and creative thinking among the learners for establishing competency in mathematical reasoning, argumentation, and problem solving.

NCF 2005 suggests that teacher should be a facilitator and be a scaffolding agent. The quality of teaching must therefore be thought of dynamically, as a function of contextual shifts. Mathematics is misunderstood and there is common among learners and even teachers some sort of anxiety which is a complex emotional response, often unconscious in origin, with fear or dread as its most notable characteristic (Page and Thomas, 1979). If such a form of fear happens or feels in connection with Mathematics, then it can be termed as Mathematics anxiety. Ashcraft and Kirk (2001) revealed that mathematics anxiety affects badly the performance. Haylock (1986) revealed that 26% of pupils aged 10 to 11 years who were low achievers in mathematics were showing an increased level of mathematics anxiety.

Another important variable related to mathematics learning is problem solving, a mental process that involves discovering, analyzing and solving problems. The ultimate goal of problem-solving is to overcome obstacles and find a solution that best resolves the issue.

Performance approach orientation is also one significant variable that can increase the performance in any subject (Lin, McKeachie, and Kim, 2003). Performance orientation is one's desire to achieve good on external indicators of success. Performance orientation is also associated with higher states of anxiety. This is achieved through mastering new situations and skills, understanding tasks, and using self-referenced standards of improvement (Elliot & Sheldon, 1997).

Besides these variables, motivation has been found to be associated negatively with mathematics anxiety. It is the internal mental state of a person which relates to the initiation, direction, persistence, intensity, and termination of behavior (Landy and Becker, 1987). The present study aims to look at the relationship between these variables among the pre-service teachers, who would be teaching Mathematics to primary and elementary classes.

Need/ Background of the Study

Statistics reveal that Delhi has an utmost scarcity of Mathematics teachers in elementary schools. The entry level survey of recent years informs that students who have opted mathematics in their senior secondary level are very few among the Diploma in ETE of Jamia Millia Islamia.

Is it because of anxiety? Are student teachers ready to learn and teach mathematics? What sort of motivation they have in mathematics? Is their performance approach orientation related to their mathematics awareness? To what extent the pre-service teachers of Delhi are good in problem solving? How much and to what extent motivation and mathematics anxiety are related?

Research survey of related literature couldn't give a good research about all these aspects. Since these problems and issues are of serious concern the researchers have designed such a study.

The Problem

The study is entitled as "*A study of mathematics anxiety vis-à-vis motivation, performance approach orientation, problem solving and impact of pedagogical intervention among pre-service teachers of Delhi.*"

Objectives

The present study is designed

1. To assess the relationship between mathematics anxiety and:
    1. Motivation in mathematics
    2. Performance approach orientation
    3. Problem solving
    4. Impact of pedagogical intervention
2. To compare male and female pre-service teachers vis-à-vis mathematics anxiety, motivation in mathematics, performance approach orientation and problem solving
3. To assess the effectiveness of the intervention (hands on experience) among pre service teachers on learning a concept of mathematics.

Method of the Study

Design

The study is a quasi-experimental study with two group post test design. The authors have selected normative survey method to collect the data.

Sampling

Sampling technique was incidental in nature. 46 pre-service teachers were selected from Diploma in ETE students from Jamia Millia Islamia.

Measures

The following tools were employed for data collection.

1. A Data Blank developed by the authors.
2. Mathematics Anxiety Scale Survey Form (Bai, et al., 2009): Chronbach's Alpha obtained for the scale is 0.801.
3. Performance approach orientation scale (Jackson, 2008): Chronbach's Alpha in reliability test is 0.782.
4. Creative Problem solving Test (Sussex Publishers): Chronbach's Alpha obtained is 0.711.
5. Modified version of elementary school motivation scale (Guay, et al., 2010): The Chronbach's Alpha is 0.734.
6. Test on fractions: the test from the content area 'Fractions'. Developed by the researchers

Intervention

The intervention is based on the course material of AMT. Diploma in Elementary Teacher Education is a two year course at Jamia Millia Islamia to prepare teachers for primary and elementary schools, besides the core and elective subjects they have to opt for at least two options of work education from the eight options available. Paper work is one of the options, the curriculum of paper work is designed in a manner so that pre service teachers besides being engaged in creative activities learn concepts of mathematics through paper work (hands on experiences), and predominant among these are fractions and decimals. However it is ensured that all operations are chosen, in this manner they learn the concept by, chalk and talk, one sub concept through hands on and by seeing others working on other concepts. Pre service teachers while learning these concepts are encouraged to illustrate the concepts through drawing; each one prepares a colorful visual with hand made paper of the sub concept of fraction/decimal on the basis of their choice, Due to time constrains each pre service teacher gets to prepare just one eco friendly and user friendly visual using colorful hand made paper. Visuals are of a size which have feasibility of carrying and storing them, they are durable because the paper is strengthened using cello tape. They are displayed on the black board using reusable glue called Blu tack. Purpose of getting these visuals prepared is to help them learn concepts which can be used during the internship and in teaching.

Statistical Treatments

PASW statistics 18th version will be used to calculate mean, standard deviation, product moment correlation and t-test.

Analysis of the Results

The analysis of the results reveal that Mathematics anxiety and motivation in mathematics are found negatively related (r=-0.39). It is meant that those who have less motivation in learning mathematics have higher levels of mathematics anxiety and vice-versa. The relationship between mathematics anxiety and performance approach orientation is found to be low but not significant. The relationship of mathematics anxiety with problem solving as well as with test on fractions is found to be negligible and insignificant.

In problem solving the male pre-service teachers show significantly higher mean score (t=3.46; P<0.01).

Pre-service teachers who have opted paper work show higher mean score on test on fractions than those who have opted other work education options (t= 2.99; P<0.01). In mathematical anxiety, motivation, performance approach orientation and problem solving both the groups show more or less similar scores.

Discussion of the Results

The results show that pre-service teachers of Delhi have an average mathematics anxiety, high motivation, high performance approach orientation, average problem solving ability and low score in test on fractions.

1. It is evident from the further results that mathematics anxiety and motivation in mathematics or pre-service teachers of Delhi are negatively related, which means that an increase in motivation will cause for a corresponding decrease in mathematics anxiety.
2. The findings of the study reveal that male pre-ser-

vice teachers of Delhi are better in problem solving than female pre-service teachers. It may be because of their increased involvement in social situation. Female pre-service teachers are confined to activities at home besides schooling, while male counterparts engage in social world. That may bring various skills to identify situations and to deal effectively when they face problems.

3. Pre-service teachers of Delhi who have opted paper work in work education got higher score in their test on fractions. It could be due to conceptual understanding in mathematics developed during intervention via paper work. When we look into details it is clear that though pre-service teachers from paper work option show comparatively higher score, the mean score of the group is still very low (2.81 out of 14). Many of them successfully have done the questions on which they had hands on experiences in their paper work session; for the rest they couldn't answer properly. It is thus inferred that hands on experiences in basic mathematics is very useful in developing conceptual clarity on fractions not only it works for children but for pre-service teachers/adults as well.
   It can thus be inferred that hands on experiences were found to be effective in conceptual understanding; nevertheless the teaching for conceptual understanding did bring overall improvement in their performance but was not satisfactory. The study clearly reiterates the previous studies (Stipek, 2002; Reynolds and Muijs, 1999), (Ali S 2004), which inform effectiveness of hands on experiences. Eventually hands on experiences will help in developing sufficient motivation among students which would facilitate diminution in mathematics anxiety. The study supports Khoush Bakht and Kayyer (2005), Md. Yunus and Wan (2009) and Middleton and Spanias (1999).

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Connected Math: Teaching Trigonometry

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A girl in Class 11 when asked to describe sine answered, "Last year, it was a ratio of sides of a triangle, this year in calculus class it is a function and in the physics class, it is a wave". This state of mind is highly representative, given the highly disconnected way in which mathematical concepts are usually taught. In this paper, we take a look at how the ideas of trigonometry are slowly built up in the mathematics textbooks of the Kerala State, and how this foundation can be used to describe the extensions of these ideas as in Class 11.

The geometrical study of triangles begins in Class 7, where triangles are drawn according to specified dimensions. A theoretical discussion of the various observations made here leads to the discussion of congruence of triangles in Class 8. One point emphasized here is that the lengths of three sides uniquely determine a triangle, but the measures of the three angles do not. This problem is taken up in Class 9 and the ensuing discussion results in the notion of similarity. It is noted that though the sides of triangles with the same three angles may not be equal, their lengths are proportional. This is seen in another light in Class 10, that though the sizes of the angles of a triangle do not determine the absolute lengths of the sides, they do determine the ratio of the sides. The search for such ratios is taken as the starting point of trigonometry.

This development of trigonometry is a pedagogical convenience and does not reflect the historical evolution of these ideas. This history is discussed as an aside in the textbooks. The historical evolution of measurement of angles is also discussed, and it is noted that the degree measure of an angle essentially gives what fraction of the circumference of a circle the angle includes and the radian measure gives what fraction or multiple of the radius is the included arc.

Here the sine, cosine and tangent are treated as numbers representing sizes of angles or in other words, measures of an angle. The very method of definition limits their application to acute angles and this necessitates the use of multiple formulas in certain computations. For examples, if the two sides, say a and b and the included angle C of a triangle are specified, we can show that its area is $\frac{1}{2}ab \sin C$ if C is acute, $\frac{1}{2}ab \sin(180-C)^\circ$ if C is obtuse and $\frac{1}{2}ab$ if C is right. If we extend the definition of sine to angles between 90° and 180° by $\sin(180 - x) = \sin x$ and $\sin 90^\circ = 1$, then 1 three different formulas can combined into a single formula $\frac{1}{2}ab \sin C$, applicable to all types of triangles. Again, under the same specifications, the square of the third side can be computed as $a^2 + b^2 - 2ab \cos C$, if C is acute, $a^2 + b^2 + 2ab \cos C$, if C is obtuse, and $a^2 + b^2$, if C is right; and the extending the definition of cosine to angles between 90° and 180° by $\cos(180 - x) = -\cos x$ and $\cos 90^\circ = 0$ all the above formulas can be combined to the single formula $a^2 + b^2 - 2ab \cos C$.

In the chapter on Analytic Geometry in Class 10, these ideas are revisited and shown that sine and cosine measures of an angle can be realized as coordinates of a point on the upper semicircle of unit radius centered at the origin. Here, the extensions of these measures to angles between 180° and 360° are made in such a way that for points on the lower semicircle also, the coordinates are the sine and cosine values of the angle of rotation.

At a suitable stage in Class 11, we can strip the sine and cosine off their geometric context and treat them purely as functions. Actually, this leads to two pairs of functions, depending on whether $\sin x$ and $\cos x$ mean the measures of an angle of x° or x *rad*. We can introduce the current convention of writing $\sin x$ to mean $\sin x$ *rad* and similarly for the cosine. This defines a function on the set of real numbers between 0 and 2π to the set of real numbers between −1 and 1. These can be then extended to the whole set of real numbers by the definitions, $\sin(x + 2\pi) = \sin x$ and $\cos(x + 2\pi) = \cos x$. The wave forms of their graphs can then described and its uses in describing various physical phenomena such as simple harmonic motion can also be discussed.

A Memory Based Model on Enhancing Mathematics Learning

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Abstract: Memory is a wonderful trait of human beings. Now, more than ever in history, scientists are unlocking the secrets to enhancing memory. Memory is extremely important to educators, not only for them personally as they age and worry about failing memory, but, most important, for the role that memory plays in the teaching/learning process. Memory, as a concept, often is relegated to a minimal role. As noted by Caine and Caine (1997), "Many of us associate the word memory with the recall of specific dates or facts or lists of information and sets of instructions, requiring memorization and effort" (p. 41). Memory, however, goes beyond this one-dimensional aspect of learning and, rather, focuses on attending, learning, linking, remembering, and using the thousand pieces of knowledge and skills we encounter constantly. For educators, memory is the only evidence that something or anything has been learned. We consider recent discoveries in brain function for declarative and procedural memories in the context of mathematical memory. These findings suggest to us a model for certain aspects of mathematics teaching, in which developing brains of children benefit from an external agent to assist in the formation of declarative memories form procedural memories. We consider implicates of this model for classroom teaching.

Introduction

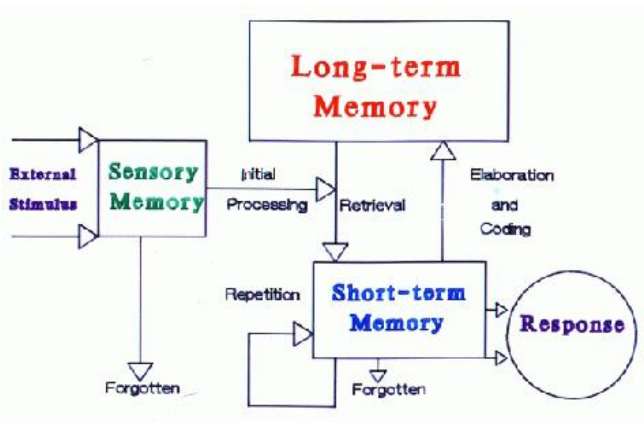
The nature of learning has been a constant area for research since the beginnings of modern psychology. Models for the activity of teaching, especially in the area of mathematics, have been much less common (see, however, Simon, 1995; Steffe and d'Ambrosio, 1995). In this article we consider recent research into how human brains process declarative and procedural knowledge. On the basis of a simple but plausible assumption on the nature of developing brains we suggest that, at least in the early years of schooling, an external agent can greatly assist in the formation of declarative memories from procedural memories in mathematics. Given the importance of both procedural and declarative memories for mathematics, this suggests a significant function for a mathematics teacher: to fulfill that of an external conduit from one part of a student's brain to another.

Memory-based Model for Teaching Mathematics

That there seem to be two distinct brain areas for procedural and declarative memory must make us suspicious. In mathematical settings, at least, the region devoted to declarative memory may have difficulty that is, few mechanisms for – taking as its basic material the activities of the region responsible for procedural memory. If so, the role of teacher becomes even more evident: as an external conduit to allow declarative memories to be formed from the raw material of stored procedural memories.

A decisive force in the creation of mathematical schema may be an appropriate agent capable of externalizing procedural memory and utilizing it so that a student can form declarative memories, both episodic and semantic. The reason for this, we hypothesize, is that the temporal and parietal neo-cortex has, in young children, few mechanisms for taking the memory activities of the basal ganglia as raw data for the formation of new declarative memories. What has to happen, we suspect, is that an agent externalizes those memories of procedures from the basal ganglia and recasts them in a form suitable for the hippocampus region and/or the temporal and parietal neo-cortex to process them as declarative memories. Our model, building on what is known about brain structures for memory, has the following features:

1. There are, in young, developing brains, relatively few direct connections between the brain regions implicated in procedural memory and those implicated in declarative memory. There are known to be direct connections into the basal ganglia – responsible for motor control and planning – from all over the brain. We postulate that in younger brains there are few direct connections from the basal ganglia to the hippocampal region and/or the temporal and parietal lobes.
2. Episodic memory, as a form of declarative memory, resides largely in the hippocampus region.
3. Semantic memory, as a form of declarative memory, resides largely in the neo-cortex, particularly the temporal and parietal lobes.
4. An external agent can assist in the formation of episodic memory from procedural memory.
5. An external agent can stimulate pre-existing connections between the hippocampus region and the co-cortex to assist in the transfer of episodic to semantic memory.



(Atkinson and Shiffrin -1968)

This model proposes that information is processed and stored in 3 stages.

Sensory Memory

Sensory memory is affiliated with the transduction of energy (change from one energy from to another). The environment makes available a variety of sources of information (light, sound, smell, heat, cold, etc.), but the brain only understands electrical energy. The body has special sensory receptor cells that transduce (change from one form of energy to another) this external energy to something the brain can understand. In the process of transduction, a memory is created. This memory is very short (less than 1/2 second for vision; about 3 seconds for hearing).It is absolutely critical that the learner attend to the information at this initial stage in order to transfer it to the next one. There are two major concepts for getting information into short- term memory

First, individuals are more likely to pay attention to a stimulus if it has an interesting feature. We are more likely to get an orienting response if this is present. Second, individuals are more likely to pay attention if the stimulus activates a known pattern. To the extent we have students call to mind relevant prior learning before we begin our presentations; we can take advantage of this principle.

Short-Term Memory (STM)

Short-term memory is also called working memory and relates to what we are thinking about at any given moment in time. In Freudian terms, this is conscious memory. It is created by our paying attention to an external stimulus, an internal thought, or both. It will initially last somewhere around 15 to 20 seconds unless it is repeated (called maintenance rehearsal) at which point it may be available for up to 20 minutes.

Long-Term Memory (LTM)

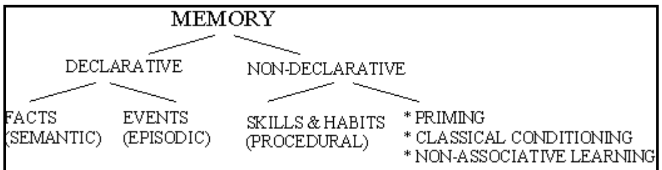
Long-term memory is also called preconscious and unconscious memory in Freudian terms. Preconscious means that the information is relatively easily recalled (although it may take several minutes or even hours) while unconscious refers to data that is not available during normal consciousness. It is preconscious memory that is the focus of cognitive psychology as it relates to long-term memory. The levels-of-processing theory, however, has provided some research that attests to the fact that we "know" more than we can easily recall. The two processes most likely to move information into long-term memory are elaboration and distributed practice. As information is stored in long-term memory, it is organized using declarative and procedural memory.

Declarative and Procedural Memory

Declarative memory (often referred to as explicit memory) is memory for facts and events, including scenes, faces and stories. Declarative memory depends upon having relevant brain structures intact: people with certain forms of amnesia, due to identifiable brain damage, lose their ability to function efficiently – or at all – with declarative memory (Squire, 1994, p.203). However, the concept of declarative memory is not due simply to dis-association of memory caused by brain damage: recent findings (for example, Eichenbaum, 1994; Ullman et al, 1997) demonstrate that specific areas of the brain are dedicated to declarative memory (see also, Bransford et al, 1999, p. 112 ff.)

Declarative memory includes episodic and semantic memory – a distinction due to Tulving (1983). This distinction is not sharp since in practice many memories involve both episodic and semantic aspects. The essential difference is that semantic memories are relatively bereft of context: they appear to the rememberer simply as known facts, whereas episodic memories involve significant components of context in which the memory was formed. The distinction in mathematical understanding is therefore critical because it relates to memories that have been pared down so as to be apparently free of context or irrelevant detail. In mathematics, episodic memories can facilitate the establishment of semantic memories (Davis, 1996).

Procedural memory is one form of non-declarative memory although the two are often identified as being the same. Cohen and Squire (1980) coined the term procedural memory for the ability to learn sensorimotor tasks in the presence of other severe memory losses. They postulated a memory for how to do things as distinct from a memory for what was done or what was recalled as a fact.



(Adapted from Squire, 1994, p. 205)

Declarative and Procedural Memory in Mathematics

The distinction that is commonly made between declarative and non-declarative memory is particularly pertinent to learning mathematics. We take it to be the case that declarative memory – at least certain forms of it – in mathematics is a higher and more useful form of memory than procedural. This is because, in mathematics, we see declarative memory (memory that) as a form of extraction or compression of the essence of a procedure or procedures. Declarative memories in mathematics seem to associated with the formation of mathematical schema and the processes of encapsulation of procedures (Tall et al, 2000). Indeed the original ancient Greek meaning was the "essence" of a thing: its characteristic nature.

A prime example of declarative memory can be observed in Maher and Speiser's (1996) account of a young student who related binomial expansions to memories of building block towers. In using her memories of building block towers to establish a potent link between these towers and binomial expressions this student was able to declare that the binomial expressions were "just like" the towers. She effectively stopped carrying out the procedure of expanding the binomial expression and declared that it was just like building towers of height 3 from two colors of blocks.

Classic examples of procedural memories arise from students' engagement with taught algorithms such as long division, multiplication of whole numbers, and the Babylonian iterative method for finding square roots, the bisection algorithm for finding square roots, and Euclid's algorithm for the greatest common divisor of two positive integers. It is a commonplace observation that many students learn to carry out these procedures but have great difficulty talking about them in the absence of doing them.

An almost universal example of a non-taught procedure that is recalled procedurally very well by children as young as 3 and 4 years of age is sharing, also known as dealing or distributive counting (Hunting and Davis, 1991; Miller, 1984). However these same children are largely unable to talk about and contemplate the act of sharing as a fact (Hunting and Davis, 1991).

Implications of the Model

Our brain-based model of teacher as external agent between an individual child's procedural and declarative memories has implications for the sorts of activities that a teacher engages in to facilitate the formation of declarative memories. The most obvious implication is for what is likely not to work. Although we regard practice at procedures and algorithms as important, our model indicates that this alone will not be sufficient for all, or even a majority, of students. The ability to form declarative memories is not going to arise simply from carrying out procedures.

Children essentially acted as external agents for each other in turning procedural memories into declarative memories. One way to assist in the formation of episodic memories, focusing on aspects of episodes pertinent to understanding mathematics, is for a student to elaborate their solution procedures to problems through explanation and discussion. A student cannot talk about their solution procedures unless they have something episodic or semantic to talk about. This is because procedural memory, by its very nature as a form of non-declarative memory, is not accessible to verbal discussion (Squire, 1994). Insisting that a student talk about their solution procedures therefore forces them to bring to mind episodes in which they carried out those procedures. In other words, it forces them to establish declarative memory for procedures in the form of relevant episodes.

Conclusion

In the enhancement of mathematical memory the major issue for a teacher to focus on is assisting students to establish appropriate episodic memories of mathematical activity. In our model of teacher as external agent, there are three possible ways in which the teacher can play a part in enhancing memory. These are, assisting in the transfer of:

1. Procedural memories to episodic memories
2. Episodic memories to semantic memories
3. Procedural memories directly to semantic memories.

Procedural to semantic memory transfer would involve establishing a mathematical fact or facts from memories that involve only habit or the execution of procedures, in the absence of episodic memories. That episodic memory plays a major role in the transition of procedural memories to semantic memories. In the formation of such episodic memories a student's focus of attention is critical. This is because not everything about an episode is appropriate to recall for the purposes of obtaining a deeper understanding of the mathematics involved.

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Finding the Mathematical Ability of Farmers and Students in Solving Problems

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Abstract: This paper aims to compare solutions given by farmers and students to problems posed on area and perimeter of a rectangle. The study is made in an Indian context. Unschooled practicing farmers were able to figure out solutions for complex geometrical configurations which occur in real life. It is observed that students have difficulty to solve problems in real life situation. Meaning associated with the process of problem solving is studied. This paper elucidates the meaning associated by students and farmers in solving problems. Mathematical ability herein is conceptual understanding, application of the concept in problem solving and computation. The need for mediation in math education is highlighted as an outcome of the study.

Introduction

It has been observed from previous interaction with students that conceptual understanding in mathematics is based on their own algorithms. Data from students on area and perimeter of rectangle was collected in earlier interactions. After studying mathematics in everyday life it is observed that people apply math in ways that are practical and easy to execute a task in real life. There is a need to understand the meaning attached to the problem solving in real life situation. A group of farmers was present for 7 days. It was decided to investigate their mode of problem solving. They were observed by giving tasks on several math concepts of area, volume, fractions, estimation and measurements. Questions asked to children were poised to farmers. Similar studies conducted in Brazil seem to indicate a general pattern in thinking approaches of people.

Farmers: Twenty five farmers form Baran, Rajasthan had visited Mumbai for Seven Day Workshop to train on making Bamboo Products. The workshop was conducted from 17th December 2009 to 24th December 2009. The Farmers were making Bamboo products to earn additional income. They have farming land ranging from Five Bhigas to Ten Bhigas. They cultivate their land and grow seasonal corps. Most of the cultivated crop is for sustenance. They partly sell the produce. The farmers belonged to scheduled caste. They believe in supernatural phenomenon and believe elders. They also believe in people in power or authority. They do not question authority. One male farmer was a class 9 school dropout. One female farmer was a class 9 student wisher to pursue further studies. Two other male farmers were class 4 dropouts. Rest of them were not fomally schooled.

Students: Pilot study of Forty One students (class 7 CBSE) was conducted in a mini project. It was found that the children had misconception about area and perimeter of a rectangle. About one hundred students (class 7 CBSE) were tested during an independent study by a student as a part of product design project. Questions from this independent study and the previous study were posed to the farmers. The students have fear of exams, teachers, parents and mathematics. All children interviewed said that they love to play. Outcome of both the student groups are presented. One child from the Waldorf school was also poised with the same questions.

Method used to Study Responses

Method used is using open ended word problems, text like formal problems. All were interviewed on a one-to-one by the researcher. Farmers had no formal schooling so the researcher had to use blocks and response was videotaped. Children recorded their responses on given sheets.

Questions were asked on the concept of area if a rectangle had area and an irregular shape had area the farmers believed that only land had area and area of irregular surface is irrelevant. All farmers knew area of a field and that field is always a rectangle. Only one farmer knew that area in formal math is S1XS2. 72% of the farmers knew that walking around the field and adding the lengths is the perimeter of the field. 84% of the farmers knew that rectangles with same area there can have different perimeters and rectangles with same perimeter there can have different areas. All farmers said they could sit, jump onto, run, go onto walk in area but they said they could not eat an area. 88% farmers said they could add subtract and divide area. All farmers said they could fill increase and decrease area. All farmers said that they did not know if they could arrange or multiply area. All farmers agreed that perimeter is not the length of fencing wire needed to fence the field. They said that formal school math would not solve the problem of estimation of fencing wire. They knew that the boundaries of the field needed posts around the field on which the fencing wire had to be turned around. They also said that posts are organic with no fixed diameter. So it is not possible to estimate precisely the length of fencing wire. They also said the gate needs different length of fencing wire. They also measure with foot length the perimeter. If given a ruler they start the measurement from the ‘1’ on the ruler. They believe that ‘0’ is of no value on its own or before a number. They use arm span for measuring lengths. The distance between the middle finger tip to the elbow is equal to the length of one foot span. This is what they believe. They are aware that different people had different lengths of hand and feet. They believed in approximating this. They have the ‘Patwari’ (government official) who measures land and they have not questioned his authority and do not know how he measures with the tape; From ‘0’ on the tape or ‘1’ on the tape. They are of the opinion that formal schooling is not effective for life situations.

Hundred students of the independent survey were asked about the concept of area. All students believed that a rectangle and newspaper had area. 20% students believed that irregular shape and ball had area. 10% students did not know that ball had area. 10% students believed that sun and hand had area. 80% students said that sun and hand did not have area. 10% students said that they did not know id sun and hand area. 50% students believed that leaf had area. 30% students believed that they could sit in an area. 80% students believed that they can ‘run’ and ‘jump on’ to area. 60% students believed that they could sleep in area. 90% students believed that they can ‘gointo’ area. 50% students believe that they can eat area. All 100 students believe that they can play in area. 80% students believed that they can add, subtract and divide area. 30% students believe that area can be decreased and increased. 70% students believe that area can be filled 10% students did not know that if area can be filled. 90% students said area can be arranged. 50% students said area can be multiplied.

In another study conducted on Forty One students of class seven CBSE it was found that 31 students believed that rectangles of different perimeters cannot have same area. 8 students said that they did not know what to answer. 30 students believed that rectangles with same perimeters and different areas do not exist. 11 students did not know what to answer. 39 students said they knew what perimeter was and 36 students said they knew what area was. 22 students said they knew how to solve problems from text. When asked about the practical aspect of fencing 41 students believed that perimeter was sufficient to fence even if there were poles surrounding the field. They felt that poles were insignificant. They also believed that fencing is for one perimeter only they could not visualise that the field had to be fenced over a certain height. To find out if children from Waldorf school attempt this differently one student of class 8 was questioned on the concept of area and perimeter. This student had seen a field as a part of experiential learning. He could not visualise the field as the scale was too large. He was sure of the formula for area and perimeter. He was not able to visualise the fencing of the field problem although he had seen a field that was fenced.

Conclusion

It is observed from the study that farmers who, cultivate land though they have fair knowledge of perimeter; they are unable to estimate the fencing wire needed to fence the field. Meaning attached by farmers to the concept of area and perimeter is connected to livelihood. They do not see area as a surface property and consider area of any other object to them other than field seems to be non-exsistant. They seem to see visualise division of area in terms of sharing or loosing land. Similarly increase in area is seen in terms acquisition of land.

From the response of the students it is observed that they rely on the formula as a tool to understand the concept of the area and perimeter. The meaning attached by the students to the learning of the area and perimeter is to get through the annual examination by scoring 'good' marks or 'passing ' the examination. All 141 students in both studies, said that they could play on area. They are referring surface of the play ground.

Mediation by a facilitator or teacher is required to bridge of gap between experiential learning and formal learning. (TLP: Prof A G Rao)

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A Teaching-Learning Sequence For Integers Based On A Real Life Context : A 'Dream Mall' For Children

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Introduction

Integer is a one of the critical topics for the middle school children. It lays the foundation for learning algebra and other higher mathematical concepts. Several researches show that school students have difficulties in understanding the concepts of either negative or signed numbers and in performing procedures with them (e.g., Janvier, 1983).

The main sources for these difficulties, as suggested in the previous research are:

- The conflict between the practical meaning of magnitude or quantity associated with numbers (Fischbein, 1987; Hefendehl-Hebeker, 1991).
- The conflict between the two different meanings (an operation and a direction) of the sign '-' as in (-1) - (+2) (Janvier, 1985; Carraher, 1990).
- The perception of zero as absolute zero with nothing 'below' it (Hefendehl-Hebeker, 1991).
- The absence of a good, intuitive, familiar model which satisfy all the algebraic properties of signed numbers (Glaeser, 1981, quoted in Fischbein, 1987).

Above issues coupled with the attention paid to the procedures and learning of rules (eg -minus and minus makes plus in cases such as 5-(-6) =11) while teaching-learning of integers makes this topic more challenging for children.

In this regard, we believe that using a close to real life context for teaching Integers will allow students to acquire in-depth knowledge without resorting to memorization of generalized sign rules often used in learning integers operations. The context selected should provide intuitive basis for students to make sense of and explore the concept so that student are able to map the actions performed in the context with the mathematics of Integers. Such contexts allow students to use their knowledge of real life events as a resource for understanding abstract concepts by bridging the knowledge gained outside and within the school. Not every context is suitable to explore the related mathematics and thus the role of teacher is important in selecting the appropriate context, understanding how various aspects of the concepts are embedded in a context and how they can be made transparent to the students.

In this poster, we present a close to real life context for teaching and learning Integers. The context is of a multi-storey building (named as 'Dream Mall') with basement floors and a lift facility.

The poster highlights:

- Theoretical framework for Integers.
- Description of the 'Dream Mall' .
- Teaching-learning sequence for Integers using 'Dream Mall' as a context .

- Strengths and limitations of using 'Dream Mall' context for teaching-learning Integers.

This context was developed during a collaborative lesson planning workshop on Integers with the teachers. The 'Dream Mall' context can be used by the teachers to help children extend their prior number knowledge to Integers. The use of a real life context engages students in meaningful learning activity while understanding different senses in which integers and their operations can be interpreted and represented mathematically. The context takes into account the difficulties involved in learning of Integers by providing transition to abstract principles through exploring the context and developing understanding of different senses in which Integers can be used.

Theoretical Framework

The poster elaborates the three broad senses (Vergnaud 1982) in which Integers are interpreted.

1) As a change

Change includes increase or decrease, movement up or down (or forward and backward) or positive or negative growth (for eg: total annual sales of a company). The reference point for estimating change may not necessarily be zero.

2) As a state

For specifying the state we use integers only when it is meaningful to talk about positive and negative states (eg: Temperature of water in a freezer, number of floors above and below the ground floor). The reference point for specifying states is usually standard.

3) As relation

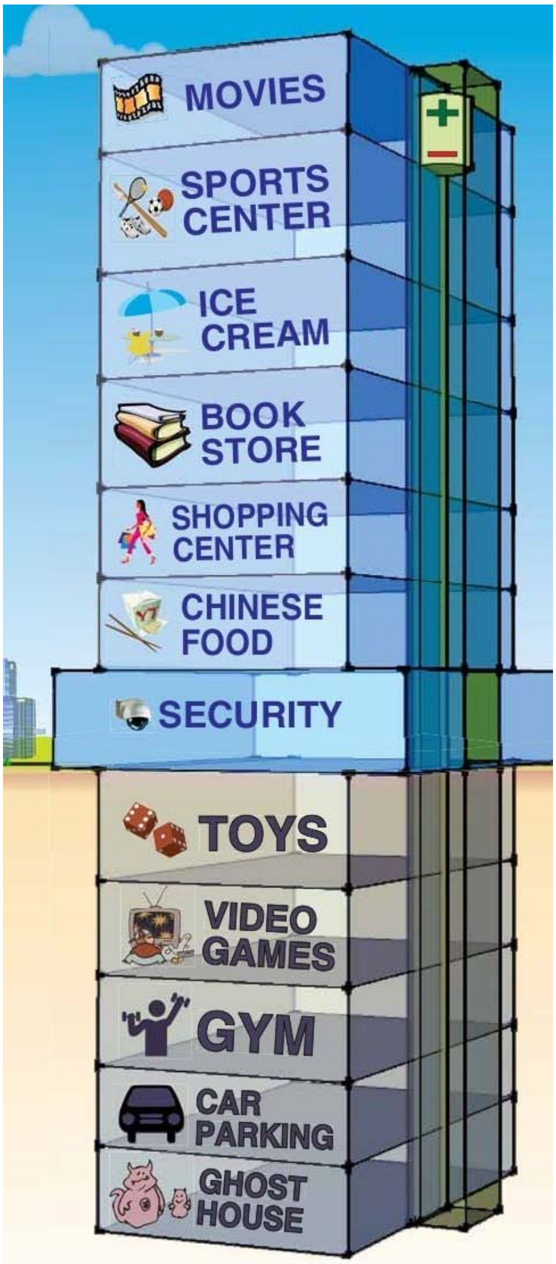
Integers can be represented as relation when there is comparison between 'states' or 'changes' with respect to the selected reference point. This is a directed relation. The relation makes sense if we distinguish the direction of the relationship and use positive and negative numbers to indicate it. (eg: Me and my sister are standing in a queue to buy ice-cream. How far is my sister from me?)

Description of the 'Dream Mall'

The 'Dream Mall' was designed with five basement floors and six top level floors. Each floor in the mall was dedicated to children fancy items like toys, video games, books.

The unique feature of the mall was the lift with two buttons labeled with '+' and '-' sign. The specially designed buttons in the lift provides a sense of direction while navigating the building. On pressing '+' button once, the lift goes one floor up. On pressing '-' once, the lift goes one floor down. Number of presses of '+' and/or '-' determines the final position of the lift in the building.

The picture of the 'Dream Mall' is shown below.



Teaching and Learning Sequence for Integers Using 'Dream Mall' as a Context

A series of questions are framed for teaching-learning of Integers in a step by step manner by using 'Dream Mall' as a context. The learning sequence for integers is described below.

1. Identifying use and need of Integers in real life

Teachers are encouraged to discuss various contexts where positive and negative numbers can be used in real life. Later, the discussions can be directed towards the context of using lifts in buildings with floors below ground level to set the stage for introducing 'Dream Mall' as a context for the students to explore.

2. Understanding Integers as a 'state'

Students can be asked to number the floors above and below the ground floor. The basement floors and the top level floors with same number (but opposite sign) are equidistant from ground level. So, the teacher can introduce negative numbers for numbering the basement floors as opposite of positive numbers and equidistant from zero.

The children are encouraged to allocate mathematical sign to conversational day to day language commonly used for indicating position or direction. For example – the floors above are associated with '+' sign while floors below with '-' sign.

3. Understanding Integer as 'directed' relationship between 'states' by using words

A sample question is provided below.

- The car parking is ___ floor(s) ____ (up/down) from the book store.

4. Understanding Integers as 'change' by representing movement across floors in words

A sample question is provided below.

- After watching a 3-D movie and you want to play video games. You will go ___ floor(s) ___ (up/down) to reach the video game parlor.

5. Forming Integer terms by learning sign coalition

The sample question can be framed asking students to keep the track of buttons pressed in the lift and finding the floor where the lift will reach.

- You pressed the lift buttons randomly in the following sequence. '+ + - + - - '. Which floor will you reach if you started on the ground floor?
- When you pressed '+ + - + - - ' you actually pressed ' +2 -1 +1 -2 ' (this is 'Mathematical form') ? Am I correct? What is the mathematical form for ' - - - + + + + + - - + + ' ?

6. Learning additive inverse by understanding sign cancellation

By pressing '+' and '-' button equal number of times , the net movement of the lift is zero as the upward (+) and downward (-) gets canceled. This can be explained to the students through questions of form as below.

- You are in a icecream parlour and you pressed '+ - + - ++ - -' . Which floor do you reach? Explain?

7. Understanding Integers as a 'change' by representing movement of lift mathematically as numerical expressions

Students can explore different ways of reaching a destination and representing them mathematically through questions such as

- Raju and Gita enters the lift from the Chinese food store. Raju wants to go to the sports center and Gita wants to go to the book store. How will they manage this ?

8. Validating mathematical equations for Integer addition and subtraction by incorporating the concept of Integer as a 'state' and Integer as a 'change'

Question of the following type can be phrased.

- Krishna's teacher told him that when he goes up from shopping center to ice cream parlour, he can write his movement mathematically as + 4-(+2) = +2. His teacher then asked him to represent his coming down from Ice cream parlour to the shopping center. Krishna's answer was : +2- (+4) = -2 . Is he right ?

9. Formulating mathematical equations for Integer addition and subtraction based on given a situation

A sample question is provided below.

- You are in the car parking and you forgot your car keys in the movie hall. How will you show your movement mathematically?

10. Generalizing Integer addition and subtraction process

Teachers are encouraged to discuss the patterns observed by the students in integer addition and subtraction operations to elicit student understanding and observations. For example, the students can be motivated to articulate generalizations by asking the observations which are true for adding positive and negative numbers.

Strengths and Limitations of 'Dream Mall' Context for Teaching- Learning of Integers

Strengths

- The example serves as a powerful context to introduce integers and its various senses. Using this context avoids frequent switching between another contexts for introducing integers and its various senses.
- The context allows the students to validate their calculations and claims visually by looking at the picture of the 'Dream Mall'. This helps children to rectify or reconfirm their concepts in case of conflict between the visual and procedural answers.
- Most children in cities have an experience of malls. Therefore, unlike in case of temperature context or credit-debit context, children easily relate to this context.
- The design and structure of the problem is flexible. The number of floors and the fancy items in each floor can be customized as per the interest of the students.

- This example can be further extended to introduce vertical number line and body gestures for learning integers. The two dimensional building can be reduced to a single line with more positive and negative floor numbers. The students can use their own body as virtual number line to represent relative positioning of positive or negative numbers using the level of their hand.

Limitations

- The context may not serve to be useful for children who have never experienced building with basement floors or lifts.
- Some aspects of the context such as unique lift are hypothetical.
- The context cannot deal with large numbers as the number of floors in the mall cannot be extended unrealistically.
- It cannot be used for multiplication and division of integers

Acknowledgment

We acknowledge the role of K. Subramaniam, Shikha Takker & Shweta Naik, HBCSE, who had major contributions in the development of this activity.

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21st Century skills: Addressing Student's and Teacher's Needs

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This paper aims to understand how the 21st century skills can be attained, from both the student's and the teacher's perspective. The 21st century skills consist of four Cs – critical thinking skills, collaboration, communication and creativity. Most education boards across countries have recognized the importance of 21st century skills ever since it was proposed by UNESCO. There is an abundance of literature available that emphasizes how to inculcate these skills in students. However, there is a dearth of instruction material that is adapted to the needs of Indian students to address the 21st century skills. We developed one such content that teaches computers and at the same time builds these skills. This paper elaborates on how the instruction material for teaching computers to primary and middle school students, covers the 21st century skills. In addition, it addresses the needs of teachers as well, because they are the delivery channels of the content. We would like to stress that unless the requirements of the teachers are addressed, the content delivery will not be effective. There is a felt need for an appropriate teacher training programme. We describe the teacher training program – topics covered, instruction strategies used. Besides, we also present empirical data on participant teachers' feedback and knowledge gained.

In India, NCERT is the central governing body for school education and most state boards borrow cues from what it prescribes. The National Curriculum Framework (2005) elaborates on how ICT education program is suited for life skills education. It also mentions that there is a felt gap for an appropriate and standard curriculum, unavailability of trained teachers and dearth of content that addresses students' education needs. in line with this view, we developed instruction material called *Computer Masti (CM)*. These instruction material addresses the 21st century skills in the following manner: (a) *Critical thinking skills:* thinking skills is portrayed by knowledge and thinking process which brings clarity of thought amongst students. Explicit teaching of step wise thinking, logical reasoning, gathering information systematically, multiple representations are included. These are applied not just in using computer applications, but real life situations and aims to develop generalizable thinking skill set. (b) *Collaboration* as a skill is presented through group activities that knowledge sharing and hence nurtures team work abilities amongst students. (c) *Communication* as a skill is presented by the use of modern day communication tools which acts as a media between individuals and groups. Some of these communication tools include multimedia presentation and e-mail. (d) *Creativity* as a 21st century skill is nurtured in CM by including activities such as digital story-telling, multimedia programming that allows students to go beyond the book and apply to a wider skill set.

If we want the students to be equipped with the 21st century skills when they graduate out of schools, we need to invest our resources in teacher training as well. Teachers have to be given not just content on what to teach, but also training on how to teach effectively. Computer teachers typically come with a technical skill set, but lack pedagogy skills. Sometimes, the situation is even more challenging since the traditional teaching style needs to alter and this requires recognizing stereotypes of teachers, disputing these, unlearning some practices, learning modern teaching strategies (e.g. Webquest, Jigsaw, multimedia scrapbook). To overcome this challenge, strategies that we want the teacher to use in the class to facilitate nurturing of 21st century skills are also used during the training. This ensures that the teacher has a first-hand experience of using these strategies and its impact.

Preliminary data on teacher feedback of training given and knowledge gained from the training explains the following. Most of the teachers mention one or a combination of the following reasons for not using innovative teaching practices: non-availability of time, pressures of completing syllabus, lack of appreciation by school administration and parents. Hence, to address these concerns, we present a training program for teachers that cover a combination of technical and pedagogy skill set.

Making Mathematics Teaching Relevant: Teacher Competencies in a Knowledge Based Society

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Abstract. Mathematics teachers' competencies are analyzed in this paper. The importance of teachers' competencies is underlined and also the importance of competencies in so called "good practices" obtaining, is studied. The definition of mathematics teachers' competencies and their taxonomy are very important in understanding the educational reform in Indian context. The recent literature and many reforms in the field of mathematics teacher education suggest that teacher preparation has a "threefold structure" with the anchoring pillars being Subject Matter Knowledge (SMK), Pedagogical Knowledge (PK) and Pedagogical Content Knowledge (PCK). Mathematics education programs should pay more attention into the learning of mathematics in social and technological context. The reform in the educational field has to pay a special attention on mathematics teachers 'competencies.

The important thing in mathematics is not so much to obtain new facts as to discover new ways of thinking about them.

- Sir William Henry Bragg

The above quote in its simplest form attempts to summarize the general expectations of mathematics learning process.

In India, mathematics occupies a high profile in the secondary school curriculum. There are a number of fundamental reasons for this. Mathematics is a language that helps us describe ideas and relationships drawn from our environment. As the mathematics of patterns, mathematics enables us to make the invisible visible, thereby solving problems that would be impossible otherwise.

Mathematics is not only used as a computational aid, but as a tool of mathematics and technology, enabling scientists to explore concepts with idealised models before utilising them in the real world.

Because of its importance, the Indian government is committed to ensuring the provision of high quality mathematics education. In this respect, a lot needs to be done. Of major concern are the consistently low achievement levels in mathematics among students at the secondary school level.

For many years the failure rate has been dramatically high in secondary level schools. This is evident in the low scores of students' Secondary school certificate (SSC) public Examination in the Mathematics subject, taken at the end of their fifth year at secondary school. For example, for the years 2010 and 2011, the percentage of those who failed mathematics was much higher than those who passed. In 2010, the percentage of those who failed was 71.3, whereas in 2011 the percentage was 75.5.

Many reasons have been advanced to explain this trend of poor performance. Schools are hit by a lack of resources such as books, equipment, and classrooms. This problem is magnified by large classes that do not allow teachers to effectively interact with students or attend to those with learning difficulties. The students' understanding is also hampered by a low level of language proficiency. In Andhra Pradesh, the medium of instruction in primary schools is Telugu, which is quite different from the vernacular language most pupils speak at home. In secondary schools the medium of instruction is changed to English. Thus, it makes it difficult for students to follow the subjects, as they do not understand the language clearly. This also affects the learning of mathematics.

Another problem is the quality of mathematics teachers. Due to the current expansion of student enrollment and the increase of secondary schools, there has been a shortage of qualified teachers. As a result, many schools have employed unqualified teachers to teach. They have also employed people with backgrounds unrelated to teaching and have failed to secure employment opportunities relevant to their courses. There are cases in schools where teachers teach mathematics even though it is not their subject of specialisation. Finally, schools also employ under-qualified teachers such as D.Ed., who are qualified to teach at the primary school level only. Moreover, those who are qualified to teach at this level of education, have significant problems due to the poor teaching preparations they received in college. The majority of these teachers lack substantial subject matter knowledge, the knowledge of what to teach, and how to teach the subject matter effectively (Pedagogy). Subject matter knowledge and pedagogical knowledge blend to form what is referred to as pedagogical content knowledge. Pedagogical content knowledge (PCK) is the knowledge of how to transform formal subject matter knowledge into something appropriate for a particular group of students.

Because of pedagogical content knowledge problems, as well as poor classroom conditions, there is an urgent need for comprehensive teacher support programmes to improve the quality of teaching. This is especially necessary in view of students' poor performance levels.

Although many studies demonstrate that teachers' mathematical knowledge helps support increased student achievement, the actual nature and extent of that knowledge whether it is simply basic skills at the grades they teach, or complex and professionally specific mathematical knowledge is largely unknown. Knowing mathematics for teaching demands a kind of depth and detail that goes well beyond what is needed to carry out the algorithm reliably. Teachers' performance on our knowledge for teaching questions including both common and specialized content knowledge significantly predicted the size of student gain scores.

Strong standards and quality curriculum are important. But no curriculum teaches itself, and standards do not operate independently of professionals' use of them. To implement standards and curriculum effectively, school systems depend upon the work of skilled teachers who understand the subject matter. How well teachers know mathematics is central to their capacity to use instructional materials wisely, to assess students' progress, and to make sound judgments about presentation, emphasis, and sequencing. That the quality of mathematics teaching depends on teachers' knowledge of the content should not be a surprise. Equally unsurprising is that many. teachers lack sound mathematical understanding and skill. This is to be expected because most teachers like most other adults in this country is graduates of the very system that we seek to improve. Their own opportunities to learn mathematics have been uneven, and often inadequate, just like those of their non-teaching peers. Studies over the past 15 years consistently reveal that the mathematical knowledge of many teachers is dismayingly thin.1 Invisible in this research, however, is the fact that the mathematical knowledge of most adult Indians is as weak, and often weaker. We are simply failing to reach reasonable standards of mathematical proficiency with most of our students, and those students become the next generation of adults, some of them teachers. This is a big problem, and a challenge to our desire to improve.

In accomplishing this, the study was guided by the following research questions:

- What are the teachers' general impressions of the school-based seminar and its components?
- What are the teachers' perceptions of the school-based seminar in enhancing their PCK&S in the teaching of probability?
- What are the teachers' perceptions of the school-based seminar in promoting peer collaboration among teachers?
- How do teachers perceive and use the exemplary materials?
- How do teachers' collaborate in their school context?
- What are teachers' perceptions about peer collaboration?
- What are the students' experiences with and opinions about probability lessons?

Suggestions for improvement

The major areas of improvement should include the following:

- elaborate classroom activities focusing on real life practical examples;
- experiments that can guide student to derive and verify certain principles;
- an increase in the number of suggested homework and test ideas that would help teachers assess their students' achievement;
- Teachers' notes that would help teachers get more in-depth knowledge on the subject matter about the topic.

Regarding facilitators, there was a need to strengthen their leadership role by supporting them through an orientation programme. This programme would help them develop skills in order to support their colleagues better during the teaching of probability lessons as well as being able to organise peer collaboration activities in their respective schools. In relation to the collaborative activities, it was suggested that the classroom observation forms for teachers should be skipped so as to enable teachers to choose their own focus of observation in case of conducting peer observation.

To better understand the issue of relevance of mathematics teaching, it is suggested three aspects need to be considered:

- What are we trying to do?
- How to guide teachers?
- What could be relevant teaching materials?

Or to express this in an alternative manner, the relevance of mathematics needs to embrace:

a) a relevant mathematics education philosophy;
b) a relevant curriculum;
c) relevant teaching approaches to the teaching of mathematics in schools;
d) relevant assessment and evaluation strategies;
e) relevant professional development for teachers.

A Shift of Emphasis

There is a need for a shift of emphasis tin the teaching of mathematics. The shift is from learning mathematics as a body of knowledge to promoting the educational skills to be acquired through the subject of mathematics. And as attempts to gain 'education through mathematics' simply by gaining knowledge are shown to be unsuccessful, the approach needs to shift from one bound by subject chapter headings, or sections to one which more closely relates to the issues and concern within society. Also, to ensure relevance of the conceptual learning within mathematics for social issues, there needs to be a shift from an introduction of the issue followed by the conceptual learning towards the interacting with the issue in a social context and then, as an important step, making use of the conceptual mathematics that is being learned to arrive at a socio-scientific decision.

The shift from conceptual learning within a subject context to conceptual learning in a social context, and which leads to socio-scientific decision making, can be illustrated by converting a concept map into a form more closely linked to the teaching and into a form which incorporates the socio-scientific decision making. This shift is promoted as a major attempt to move to more relevant mathematics teaching in the eyes of students.

The Teaching Approach for Relevance follows the reformed map

Teaching geared to the goals of education covers a wide range of intended targets in the intellectual, personal and social domains. Conceptual learning within the subject needs to be approached in a relevant manner, but also the teaching must not lose sight of the fact that the attitudes, communication abilities and personal attributes (such as creativity, initiative, safe working) need to be developed. Strongly this suggests a societal beginning for the teaching approach. And by also encouraging student involvement, a teaching approach that builds on prior constructs held by students, thus enhancing relevance in the eyes of students.

The suggestion is that the teaching of a sequence of mathematics lessons begins from a relevant socio-scientific context. The teaching progresses from the societal (the familiar), to the mathematics concepts (the unknown), which are needed to better appreciate the issues, or concerns, and then proceeds to the socio-scientific decision making needed (the purposeful learning involving all educational domains). Teachers need to recognise that curricula promoting mathematics fundamentals, grouping mathematics concepts for scientific convenience, rather than for popularity is not the approach to promote education through mathematics. Such an approach leads to an academically perceived course that is likely to be abstract, difficult ad irrelevant.

The mathematics teacher training program should offer:

- clarity of the objectives and whether they are achievable;
- validity of the subject matter content;
- practicality and validity of the formulated activities;
- validity of the test questions and suggested homework ideas;
- general format of the materials;
- Whether the lessons gave enough guidelines for the teacher to conduct activity based lessons.

A mathematics teacher should know how to relate mathematics with other subjects and to frame phenomena in a multidisciplinary context. Mathematics teachers who teach one discipline should be able to relate its content to relevant content in other disciplines (inter-disciplinarity) or in disciplines such as science or technology (multi-disciplinarity). Phenomena by their nature involve various disciplines: so a teacher should be able to search for explanations involving the single discipline and at same time frame it in a multidisciplinary scenario.

Therefore, a mathematics teacher training program should offer trainees the knowledge and awareness of the relationships between disciplines. This competency cannot be accomplished in the framework of a strict disciplinary-separate training program. Furthermore, prospective teachers should be provided with instruction that facilitates the identification and development of concepts that unify the traditional mathematics disciplines. In such a training there should be included specific learning opportunities and instruction that would help prospective teachers to develop such interrelationships Mathematics education programs should pay more attention into the learning of mathematics in social and technological context, such as field trips, arranged visits to museums or to industries and institutions .Such training programs must allow teachers to develop a deep understanding of scientific ideas and the manner in which they were formulated.

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Supporting School Mathematics Education through Open Educational Resources

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Abstract: Homi Bhabha Centre for Science Education (HBCSE), a constituent unit of the Tata Institute of Fundamental Research (TIFR), Mumbai has launched an ambitious programme to design and test Open Educational Resources for Schools (OER4S) in school mathematics. These are web-based resources developed collaboratively by academicians, teachers and enthusiastic parents to support the learning of school mathematics. They are uploaded to the specially developed website (www.mkcl.org/mahadnyan). Special feature of the material is that it is developed in regional language (Marathi) and is made available to the stakeholders through the distributed classrooms set up by the Maharashtra Knowledge Corporation Limited (MKCL). An attempt is being made to field-test the suitability of the material for school children, their teachers and parents in the state of Maharashtra. The philosophy of the programme, the strategy of its implementation, mechanism of obtaining feedback from stakeholders for mid-course corrections are described in this paper.

Introduction

A large number of students in India attend schools that teach in regional language. In the state of Maharashtra more than two thirds of the students study in Marathi (the language of the state of Maharashtra). A majority of these students comes from rural and educationally backward communities. Moreover, these students study in schools run by local self government organizations (like zilla parishad and municipalities) that have meager educational facilities. The supporting educational material available in regional languages is negligible. As a result, they have to depend on classroom instruction and terxt book for the learning of school mathematics which is so crucial for the life of a common man. With a view to overcome this problem it has been decided to develop suitable support material and make available free of cost on the specially designed website to improve learning an teacing of school mathematics.

The Project

The main objective of the project is to make available resources that can be used to improve the quality of school mathematics education. The project is being funded by the Rajiv Gandhi Science and Technology Commission of the Government of Maharashtra and is being implemented jointly by the Homi Bhabha Centre for Science Education (HBCSE), Maharashtra Knowledge Corporation Limited (MKCL) and the Indian Consortium for Educational Transformation (I-CONSENT). This material is made uploaded to the website (www.mkcl.org/mahadnyan) specially designed by MKCL for the project. Stakeholders of education (students, teachers and parents) are encouraged to register at this website and visit it regularly. To facilitate reaching of the material to stakeholders an attempt is made to take the help of distributed classrooms set up by MKCL all over the state. The stakeholders are expected to visit these classrroms, go through the material, use them and provide us feedback for modification. The project has three components: Material development, uploading to the website and field testing. It would be informative to look at each of these aspects critically.

Material Development

Over the past three decades HBCSE has developed methods and materials to provide quality education in schools. Most of this material has been tested through its field projects undertaken both in rural as well as in urban areas of the country. Part of the open educational resources for the present project is drawn from the pool of material that HBCSE already has in its stock. Apart from HBCSE there are a large numbers of organizations working for the improvement of school education. Those organizations are approached to get their resources. In addition, special resource generation workshops are organized periodically to prepare material taking into account the needs and requirements of students, teachers and parents associated with school education. It would be appropriate to discuss a few important features of these workshops.

Participants for the workshops are chosen from the pool of innovative school teachers, teacher educators, social workers and enthusiastic parents. The participants are given the idea of open educational resources and are asked to prepare material suitable for students, teachers and parents. Material, thus, developed is processed at HBCSE and the printout is sent to the author for revision. Once the revision is incorporated the material is subjected to quality assurance. Suggestions received from Quality Assurance Teams are incorporated and the unit is finalised.

Uploading to the Website

The approved unit is first tagged for uploading to the website. It involves classifying the material in three categories: Students, Teachers and Parents. It is then suitably divided into three levels: primary (catering to grades 1 to 5), upper primary (catering to grades 6 to 8) and secondary (catering to grades 9 and 10) to enable the end users to locate the material they want.In addition, there is some material that is of general nature and is useful to all the stakeholdes. It is put as a common material. It would be informative to know the nature of this material.

Material for Students

The entire course covered at school level is divided into 48 units: 9 units for primary, 19 units for upprr primary and 20 units for secondary level. Each unit focuses on one main concept from matematics curriculum like set theory, surds, etc. The unit is further divided into subunits discussing subconcepts in it. The matter in each subunit falls in one of the following categories:

1. Content enrichment for providing additional relevant information to the students relataed to school mathematics bringing out its relavnce to day to day life.
2. Puzzles and to keep students engaged in academic activity and to develop positive attitude towards school matheamtics.
3. Anecdotes from the lives of mathematicians to motivate students to pursue mathematics education.
4. Simple experiments and Projects that can be undertaken by the studnets in their spare time.
5. Questionnaires for revising the content and for self assessment
6. Remedial and practice exercises for improving problem solving skills

Material for Teachers

HBCSE has been conducting in-service training courses for practising mathematics teachers for the last three decades. Through these courses it could gain insights into the needs and requirements of teachers teaching mathematics in different set ups. Based on these insights following types of material is developed for teachers.

1. Conceptual Discussion that consists of a. Concept Maps, b. Lesson Plans and c. Explanatory Notes.
2. Teaching Aids which has a. Power Point Presentations, b. Models/Posters, c. Guidelines for making low cost teaching aids in schools
3. Experiment/Activity/Projects comprising a. Concept-based experiments, b. Simple activities and c. Projects that can be completed in a short time.
4. Pedagogic Guidelines focusing on a. Learning difficulties, b. Recent Developments in Educational Psychology, and c. Classroom management
5. Research and Innovation providing information on a. Review of research mathematics education, b. Innovative Practices in Mathematics Teaching, c. Guidance for undertaking Action Research with illustative examples.
6. Assessing Students' Learning encompassing a. Diagnostic Testing, b. Cognitive and Affective Assessment and c. Outcome based Assessment

Material for Parents

Since a large number of parents in India take interest in the education of their wards the website material is planned taking into account their needs and requirements. It is essentially of seven types:

1. Everyday mathematics highlighting the role of mathetics in day to day life.
2. Changing parenthood bringing out the cahllenges of 21st century and changed expecetaions from parents.
3. Learning-teaching process explaining the concepts of teaching and learning and suggesting ways to faciliate them.
4. Out of school activities to support school mathematics education providing guidelines to parents how can tey augment the learning by their wards.
5. Identification and nurturance of mathematical talent at an early stage.

Common Material

The common material is meant for all the three skaholders (students, teachers and parents). It has five sections that refer to the following:

1. Question-answer section involving answers to questions related to school mathematics raised by school students.
2. Biographies of known and unknown Mathematicians who have contributed to the development of the discipline.
3. Books and articles that have been published elsewhere but are useful for students, teachers as well as for parents.
4. Open forum for free dialogue between stakeholders and experts. It also informs various stakehold-

ers about conferences, seminars, workshops, contests, etc. related to school education.

Field Trials

An attempt is made to try out the OER material in the school system. For this purpose two big educational societies from the state of Maharashtra have been identified that run schools in Marathi medium and cater mainly to students from lower socioeconomic strata. Mathematics teachers from these organizations were invited to participate in a three day workshop held at HBCSE. During this workshop teachers were familiarized with the open educational material prepared for students, teachers and parents. They were encouraged to visit website regularly, make use of the material in their day to day teaching. In addition, it was suggested that they encourage students and parents to visit the website and look for material prepared for them. These teachers were finally appealed to give us feedback based on their first hand experiences gained in using the OER for supporting mathematics education.

In general, teachers appreciated the plan of providing supporting material to the students, teachers as well as parents and felt that it would certainly bring about qualitative changes in the teaching and learning of school matheatics. Feedback is collected through email. postal correspondence, telephonic conversation and personal interaction. Based on the feedback received appropriate modifications are being made in the material. In addaition new material is being prepared and uploaded taking into account the request from students, teachers and parents. OER has thus become an ongoing activity at HBCSE.

Summary and Implications

E-learning as stated by Zemsky and Massey (2005) has a tremendous potential to improve school education. The present web based programme offers ample opportunities to cater to the needs of students, teachers and parents from different parts of the state. It will eventually make available innovative material developed by different organizations working for the improvement of school education in the country. This effort, would hopefully, bring in qualitative changes in the classroom interaction in rural as well as in urban schools. National Knowledge Commission (2007) set up by the Government of India clearly brings out the importance of open access to information and Global Open Educational Resources. The work reported in this paper is in tune with the recommendations of the commission. There is a need for similar work developing OER in regional languages and pulling together experiences from different regions of the country.

It must be noted that the work envisaged in the proposed project is of gigantic nature. It needs the cooperation of people in content, pedagogy, technology as well as in assessment. It will bear fruits only if people with different expertise come together to develop, test and modify the material. The process is expected to be an ongoing activity where the developmental work continues for ever. Presently, there are three institutions working together (HBCS, MKCL and I-CONSENT). We have are trying to seek the help of other organizations and personnel for this activity.

The project is still underway and the work done so far has given positive feedback. Teaching community has welcomed the idea of OER material. At the same time there are encouraging responses from students and parents too. It is hoped that the work will take a concrete shape soon and will make available field-tested useful material for Indian school system. India shares common educational problems with many developing countries. It is, therefore, envisaged that the Open Educational Resources developed for India will have a global relevance to bring about qualitative improvement of teaching of mathematics at school level.

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Acknowledgements

The project referred to in this paper is funded by the Rajiv Gandhi Science and Technology Commission (RGSTC), Government of Maharashtra. It is a pleasure to thank RGSTC for its generous financial support. The project is implemented jointly by HBCSE, MKCKL and I-CONSENT. We would like to express our gratitude to the heads of these organizations for their unreserved support. We would also like to thank our colleagues at HBCSE for their untiring work to make the project successful.

Effectiveness of Training of Mathematics for Capacity Building of Teacher at Upper Primary Level

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Abstract: In this study we shall try to find the effectiveness of training that can make the teachers best in teaching mathematics. Teacher Training is one of the important component in the area of teaching. Trough teacher training the awareness level of teacher increases. The awareness area of the teaching may be competency of the content and different techniques of teaching and pedagogy. The impact of the training may bring a lot of change in the transactions in the classroom process. The training helps the teacher to think and innovate new ideas of teaching. One of the good things in the training is that teachers participate from different schools and there would be presentation by different teachers during the training. So they get ample opportunity to share their way of teaching in the classroom. This makes a great impact to the teachers and the best methods they can follow in their teaching pattern in the classroom.

The capacity building is very important part to upgrade our knowledge and skills. Some teacher may be against the training. They may feel that since they have acquired higher degrees that are why training is not required for them. Literature says that "the teacher is born not made". Speaking on the necessity of training Cavanagh (1932) says:

"Nature sets the limits; as in every other direction nurture never out weights nature. But within this limit there is room for enormous improvements. Every beginner, even the most gifted, makes all sorts of mistakes which can be immediately pointed out and immediately corrected by a supervisor. Yet if a man is not trained he may never have the chance of friendly criticism."

There is different view in different literature. Lindsey (1961) says "An individual who qualifies as a professional of whom he is a part is a liberally educated person. Possesses a body of specialized skills and knowledge related to an essential for the performance of his function and is able to make rational judgments and to take appropriate action within the scope of his activities, and is responsible for the consequences of his judgments and action."

Teacher's places primary emphasis upon his service to society rather than upon his personal gain. Actively participates with his colleagues in developing and enforcing standards fundamental to continuous improvement of his profession and abides by those standards in his own practice. Practice his profession on full time basis and is engaged in continuous search for new knowledge and skill. Accordingly Lindsey, (1961) some aspects of professional personal are as follows:

(i) The Educator must make is how to use basic education principles relating to the nature of the learner and the learning process in wide range of situation.
(ii) How to use basic education educational principles relating to the learner and the learning process in a wide range of situation.
(iii) Teaching methodology and the selection and use of instructional materials.
(iv) Becoming an intelligent consumer of educational
(v) Becoming a responsible member of the teaching professional
(vi) Continuing preparation for intelligence participation in community life as a spokesman for education

In the same context it has been said there that the professional educator is a person committed to moral and intellectual excellence, to teaching scholarship. That is to be fitted for trusteeship; a person must be captured by an ideal, a self-image committed to moral and intellectual excellence. The needed image is that of a teaching scholar who: Has a genuine interest in learning and continuing to learn. Has the urge to share knowledge in ways to help others, in turn, to develop competence and genuine interest in learning. According to D'souza and Chatterjee (1956), the following are objectives of professional training: To equip teachers with essential professional tools –those professional skills and concepts required by all the teachers. To give teachers a sympathetic understanding of the physical, mental and social characteristics of children at all stages of development and adults. To equip them with the essential techniques and teaching methods appropriate to their subjects and grade. To give them knowledge of the organization and management of class instructions in the type of school in which the teacher intends to teach. To give them opportunities for acquiring a safety minimum of teaching skills through observation, participation and actual teaching and supervision.

To equip the individual teacher with an integrated working philosophy of education and the contributions which he may be expected to make in his field.

Teachers are guardians of the Children. Children may not listen their parents. But they listen their teacher voice. Society must respect and honor it. The old Chinese proverb says:

If you wish to plan for one year plant grain,

If you wish to plan for ten years plant trees,

If you wish to plan for hundred years plant men.

The following questions were framed to conduct the study. The questions were given below:

(i) To explore the level of satisfaction of trainees in a teacher training programme.

(ii) To suggest the possible areas of development for effective capacity building.

Based on these research question questionnaire were developed and at the end of the training the questionnaire were given to the participating teachers. Later the data were entered in the excel sheet and analysis is carried out. It has been observed that the training was very useful to the teachers. The training motivated the teachers' lot and they talked about the necessity to change the course design and style of presentation of experts.

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Mathematical Proofs - A Pedagogical View

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Mathematics is mainly focusing to stimulate, challenge and inform students and also to develop skill of reasoning in them. The proofs of the results or theorems are also for developing logic and reasoning in students. But it should not be in a mechanical or artificial way. In most of our high school curriculum, the scope of inspiration, challenge, intuitive thinking, reasoning etc. are too little. So many mathematical results, theorems and their proofs are presented all of a sudden in a haphazard way without giving any chance for students to think, argue, and reason out. Most of the teachers are also very eager to present those results and their proofs directly in the Skelton form, which will kill all the beauty and art of reasoning hidden in the results. Also it will destroy the art and skill of reasoning in the students. In this paper I would like to throw light to how children are to be motivated and directed to reach at mathematical results or generalizations and reason out it logically. These strategies are used in the present Mathematics text books of Kerala prepared by SCERT. Mathematical topics or ideas are presented heuristically in a sequence. Justification of results is supplied if situation needs. The proof of the result will be smoothly given in logical arguments. Consider the example: "sum of 3consecutive integers is multiple of 3". This can be verified by students by taking some particular instances and from there itself they can deduce the result.

4 + 5 + 6 = 15;

7 + 8 + 9 = 24

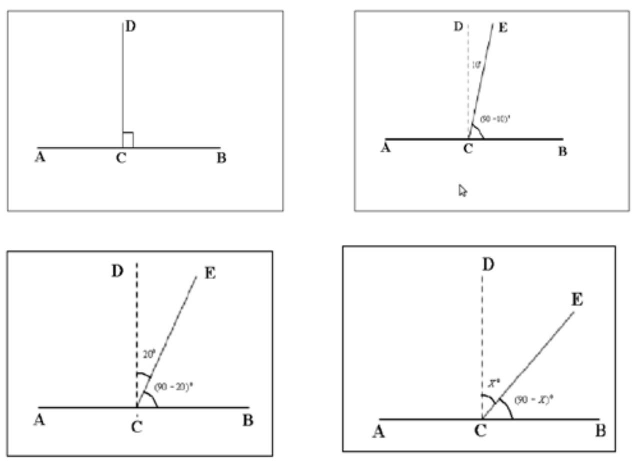
This can be re-written as (5-1) + 5+ (5+1). Here -1 and +1 can be cancelled which will be resulting in to thrice of 5.

That is (5-1) + 5 + (5+1) = 3 * 5 which is multiple of 3.

Similarly 7 + 8 + 9 = (8-1) + 8 + (8+1) = 3 * 8, multiple of 3.

If this logic is obtained for the students, then they can say generally extreme numbers are 1 less than middle one and one more than middle one, resulting their sum as multiple of 3. Now students can write this algebraically as (x-1) + x + (x+1) = 3x, always multiple of 3. Which completes the proof. From these results children can move to another result that whether sum of 3 consecutive odd numbers is multiple of 3? (or consecutive even numbers) and then to sum of 3 consecutive terms of an AP is multiple of 3 or not. Also students can think about whether sum of the five consecutive integers has similar property, and so on.

In Geometry also similar strategy can be adopted. Consider the result "linear pair is always supplementary". This can be felt by students by following sequential figures



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n general sum of two angles of a linear pair can be expressed as (90+x)+(90-x)=1800, which is very simple from the above figures. This type of reaching at conclusion can be called as logical induction. Also results will be stated only after the analysis of the situation and reaching logically at some conclusions naturally is preferable, which is usually lacking in most of our text books and teaching practices. For example suppose students are aware of segments in a circle and the result "all angles in a same segment are equal". Instead of directly stating 'angles in alternate segments of a circle are supplementary', and explaining its proofs, it is better to create curious thought in children by putting question "whether there is any relationship between angle in a segment and angle in its alternate segment?". Here children can reason in the following way – angles in a segment are half of central angle of an arc and angles in the alternate segment are half of the central angle of the alternate arc (remaining arc) of the circle. This directly gives rise to sum of the angles in a segment and its alternate segment is supplementary. Here only we are stating the result. This way of heuristically and logically arriving at results and then stating the results will be more natural for students to learn. This approach also helps them to attack and arrive at new hypotheses and results independently.

Hidden Cultural and Historical Variables to Promote Mathematics and Mathematics Education – Are There Royal Roads?

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Abstract: In a classroom if we ask the students the subject which they find it uninteresting one, most of them say Maths. They get knowledge in parts so it is not very meaningful and they do not appreciate it. The students do not achieve a broad perspective of the subject The teachers are transacting the curriculum as prescribed. And that's it! We are producing students who learn the same concepts prescribed in the syllabus by rote memorization and score good marks. But they are not aware of from where these concepts came up. They may know the formulae or the definitions but they are not aware of who was the one who thought of it.

The answer lies in the quote given below.

One can invent mathematics without knowing much of its history .One can use mathematics without knowing much,if any ,of its history. But one cannot have a mature appreciation without a substantial knowledge of its history.
-Abe Shenitzer

The researchers of this study would like to take a step more,

If you want to make an impact on someone such that he develops an appreciation for the subject then pass it through the hidden curriculum such that he himself is not aware of the impact.

That's the essence of this study. It will employ such an approach of transmission of historical and cultural aspects of Maths through different co curricular activities for the pre-service teachers. The study thus tries to ascertain the impact of this approach on the appreciation and attitude of pre service teachers.

Introduction

Mathematics has always been a subject disliked by most of the students. The importance of this subject is not felt as it is taught in isolation from the other subjects. As a result the students do not appreciate the contribution of this subject to life. We need the students to develop a sense of appreciation towards the subject, which is at the core of all the subjects. This can be possible if their teachers itself would appreciate the subject. So the pre-service teachers in the teacher training institutes need to have the appreciation towards the subject Maths first .This would expect the teachers to focus on the domains other than cognitive and psychomotor i. e affective domain. Aiming towards it the author felt that the appreciation could be developed in the would be teachers through the inclusion of the cultural and historical aspects of mathematics education in the pre service teacher training program.

Though there is introduction of new modes of classroom interaction during teacher education programs, traditional textbook and teacher-directed approaches prevail in mathematics classrooms. According to several researchers (Jaworski & Gellert, 2003; Lerman, 2005), such approaches still dominate because of a number of socio-cultural issues relating to classroom culture, the perceived nature of mathematics, acceptable styles of interaction, and personal epistemological beliefs.

As this paper attempts to find out the impact of incorporating the cultural and historical components in the teaching of mathematics for the pre-service teachers, the author is in the process of interweaving mathematical ideas into the cultural components of our country. She was interested in exploring how mathematics can be viewed culturally through the teaching of Indian Studies.

Also the historical approach of dealing with Maths teaching needed to be focused. Though in the Indian textbooks we can see the information about the mathematicians, but not much of weightage is given when it comes to class teaching. We need to incorporate the valuable contributions of Mathematicians in our teaching of Maths. This is possible only if we make them aware at the pre-service training course about the contributions of the mathematicians. Thus it has been incorporated rightly in the B.Ed. curriculum. However the author thought instead of transacting information and passing to the pre service teacher they should be given this information through the co curricular activities which would make a greater impact.

Go down deep enough into anything and you will find mathematics. ***- Dean Schlicter***

Background and theoretical framework

One of the most significant areas of research development in the last 2 decades has been that of ethnomathematics (see Ascher, 1981; Gerdes, 1995). It has not only generated a great deal of interesting evidence, but it has fundamentally changed many of our ideas and constructs. The most significant influences have been in relation to:

- Human Interactions: Ethnomathematics concerns mathematical activities and practices in society, which take place outside school, and it thereby draws attention to the roles which people other than teachers and learners play in mathematics education.
- Values and Beliefs: Ethnomathematics makes us realise that any mathematical activity involves values, beliefs and personal choices.
- Interactions between Mathematics and Languages: Languages act as the principal carriers of mathematical ideas and values in different cultures.
- Cultural Roots: Ethnomathematics is making us more aware of the cultural starting points and histories of mathematical development.

In general, these points have forced mathematics educators into giving more consideration to the overall structure of the mathematics curriculum and to how it responds, or more usually how it does not respond, to the challenge of culturally based knowledge. In general, as was said above, the mathematics curricula which exist in the diverse countries of the world do not appear to be culturally responsive in that they are remarkably similar.

It is clear that the concept of culture can be taken to have not just a macro- meaning but also a meso- meaning and the micro- meaning. If we assume that cultural influences work at the meso-level of schools as educational institutions, then we can see for example that the curriculum structures have generally evolved to suit the preparation of an elite minority of students who will study mathematics at university. If we then consider the majority of school pupils who either never go on to study more mathematics or who don't even go to university, this elitist mathematics education can be thought of as highly inappropriate. It contributes significantly to the widespread problems of alienation felt by many students towards mathematics in particular and also towards schooling in general.

Research therefore needs to explore how the mathematics curriculum can be made more culturally responsive, in order to encourage more participation at the higher levels particularly amongst cultural minority groups, where culture can imply not just ethnicity, but also socio-economic and religious diversity, and physically and mentally challenged students.

Aims of this study

- To promote a positive attitude to Maths so the it is seen as an enjoyable and interesting subject.
- To help the pre-service teachers to correlate History with Maths .
- To heighten awareness of the use of Maths in everyday life and the world outside.
- To foster an appreciation of cultural and historical aspect of maths and developing the ability to identify the relation of Maths with culture
- On the basis of above aims of the study the author expects to answer the following research questions
- Does the hidden curriculum through the organization of co curricular activities enable to develop a sense of appreciation in the pre-service teachers?
- To what extent do the pre-service teachers show a favourable attitude in incorporating the cultural and historical elements of Maths in their classroom teaching?

To answer the above research questions the author focussed on the infusion of the core elements given by NPE 1986 through the co -curricular activities from mathematical point of view.

Variables

Hidden cultural and historical variables: Cultural and historical variables includes co curricular activities based on the contributions of great mathematicians, the festivals of different cultures, the architecture of ancient times ,the occupations of people in different cultures,the textile printing in different cultures

Sample

A group of 25 pre-service teachers doing the B.Ed. program at the K. J. Somaiya Comprehensive College of Education ,Training and Research was taken as the sample according to convenient sampling technique

Participants

The author focused on celebration of different festivals throughout the year including activities in relation to mathematical concepts. The task of organizing the cultural festivals throughout the year from mathematical point of view was given to the members of Mathematics club. The Mathematics club members discussed together and came out with different ideas of organizing these festivals keeping the focus on the basic concepts in maths. Thus the concepts of congruence, ratio and proportion, geometrical shapes etc were used in different competitions like Twin competition, Sheer Korma Competition, Mathematical Mehndi & Rangoli competitions respectively. Some of the amazing Rangolis were seen at the Diwali Rangoli Competition which included Lord Ganesha using different geometrical shapes.

Also as an activity of Mathematics club, the pre-service teachers organized an Exhibition where they prepared collage and posters based on different pictures like the testellations seen in ancient architechture, the baskets made by the African tribal people and many other posters which projected the relation of culture and mathematics.Mathematics definitely has its roots in different cultures is what was concluded in the organization of co curricular activities.

History of Maths was also incorporated in the pre service training curriculum through the activity of making Computer Assisted Presentations. Here each teacher was given a task of surveying and presenting the contribution of Mathematicians through the use of ICT. In the initial phase the pre service teachers surveyed the information through the resources available like the library and the internet .Then they were equipped with the basic computer skills like making simple power point presentations. Finally they were also made aware of how to make use of movie clips in their power point presentations by teaching about movie maker. Thus the pre-service teachers came up with excellent power point presentations based on the contribution of the Mathematics.

Data Collection Technique

- An open ended interview was conducted in which the pre-service teachers were asked about whether they appreciated the inclusion of relation of culture and History with Maths. Data was collected here in written form.
- Journals were maintained by the group members of Mathematics club keeping the objective clear which also included the experiences of pre-service teachers during the organization of the activities of the club.

Following conclusions can be made on the basis of collected data:

The incorporation of historical and the cultural aspect of Mathematics education definitely had made a deep impact on the affective domain of the pre-service teachers .It was amazing that all the pre-service teachers had exchanged the information collected and the power point presentations and most of them were in favour of using this in the teaching of Maths in their future classrooms.

The author reflected by reading the journals of the pre service teachers that they felt that maths can be best transmitted by including it in the co curricular activities. The research concludes that the cultural and historical aspect of Mathematics could be best transmitted through the hidden core elements which are transmitted through the organisation of co curricular activities and not by the traditional method. It was found that the interest towards this subject has tremendously increased due to this approach. The pre-service teachers felt that culture and maths can never be separated and that Maths has been at the centre of all the subjects and all the spheres of life. Thus this research gave a broad perspective of viewing this subject.

Challenges

- In this study the sample size was quite less so we cannot achieve better results unless this kind of approach is focused in all the Teacher Training Institute. Only then the future entrants I e the students of these prospective teachers would develop the appreciation of the subject Maths.
- The conditioning of the class to the traditional modes of teaching for years in the schools could make these teachers also accept the same kind of approach. Thus there may be a need for sending the existing teachers to go in for an in-service teacher training program may be in vacation where they also learn to view the subject from broad perspective. Workshops can be organised of them so that they can do such kind of activities and relate the historical and cultural aspects of Mathematics to class teaching

References

Using history of mathematics in the teaching and learning of mathematics in Singapore http://math.nie.edu.sg/wkho/Research/My publications/Math Education/hom.pdf

The introduction of history of mathematics in Norwegian schools Bjørn Smestad, Finnmark University College, Norway. http://home.hio.no/~bjorsme/taipei.pdf

Critical Issues In Researching Cultural Aspects Of Mathematics Education Alan J. Bishop, Monash University, Melbourne, Australia Alan.Bishop@education.monash.edu.au

Program Schedule

Day 1 (January 20) Friday

09:30 to 10:00	Registration
10:00 to 10:30	Inauguration
10:30 to 11:10	Introduction to ICME, NIME and the RCs (*K. Subramaniam & R. Ramanujam*)
11:10 to 11:30	Tea
11:30 to 11:55	*Shailesh Shirali & Jonaki Ghosh*: Missed Opportunities in the Higher Secondary Mathematics Curriculum
11:55 to 12:20	*Jayasree Subramanian*: Re-Visioning school mathematics curriculum to address Social Justice Concerns in India
12:20 to 13:00	Paper Presentations:
	(i) *Shah, M. et al.:* Insights into Student Errors in Ordering of Fractions, Equal Sign Interpretation and Identification of Shapes
	(ii) *Threja, A.*: Lesh's Model of Multimodal Representation in Learning Mathematical Concepts
13:00 to 14:00	Lunch
14:00 to 14:40	*R. Ramanujam*: School mathematics, the discipline of mathematics and transitions
14:40 to 15:20	*Geetha Venkataraman*: Undergraduate Mathematics Education
15:20 to 15:45	Tea
15:45 to 16:10	*M. Mahadevan*: Contribution of Teacher's Associations towards Mathematics Education
16:10 to 17:40	Reports of the Regional Conferences: *Vijay Kumar Ambat, Geetha Venkataraman, Bhaba Kumar Sarma, Vinay Kantha, Vinayak Sholapurkar*
18:00 to 19:30	Videos

Day 2 (January 21) Saturday

09:30 to 10:10	*Anita Rampal*: The Elementary Mathematics Curriculum in India
10:10 to 10:50	*Hridaykant Dewan*: Maths Education – The efforts from outside the system
10:50 to 11:10	Tea
11:10 to 11:35	*Parvin Sinclair*: Distance is not a barrier
11:35 to 13:00	Paper Presentations:
	(i) *Mihir Arjunwadkar & Abhijat Vichare*: The Making of an Academic Programme in Modeling and Simulation
	(ii) *Chanchal Yadav & Anita Rampal*: Understanding Assessment for Learning: Reflective Practice in a Primary Mathematics Classroom

	(iii) *Mohd. Mamur Ali:* Students' Solving Processes of Linear Equations at Elementary Stage
	(iv) *Jasneet Kaur*: Children's Understanding of the Geometrical Concepts: Implications for Teaching Geometry
13:00 to 14:00	Lunch
14:00 to 15:30	Poster session (Tea will be served during the poster session)
15:30 to 15:55	*E. Krishnan*: Teaching Limits
15:55 to 17:15	Paper Presentations:
	(i) *Haneet Gandhi*: Insights and Challenges in the Teaching-Learning of Probability
	(ii) *G. Sethu Madhava Rao*: Problems of Teaching & Learning Mathematics in Bilingual Settings with Reference to Andhra Pradesh
	(iii) *Satyawati Rawool*: Using Mathematical Concepts for Engaging Learners in Thinking Process
	(iv) *R. Maithreyi*: Rethinking Primary School Mathematics: Directions for a Process-oriented Curriculum
17:30 to 19:00	Videos
20:00 to 21:30	Conference Dinner

Day 3 (January 22) Sunday

09:30 to 10:10	*K. Subramaniam*: Mathematics education research across the world
10:10 to 10:50	*K. Ramasubramanian*: History of Indian Mathematics and its Implications for Mathematics Education
10:50 to 11:15	*Sujatha R.*: Math Education: Can we explore and exploit ICT?
11:15 to 11:35	Tea
11:35 to 13:00	Panel Discussion on Assessment culture and its impact on mathematics education. Panelists: *M. Prakash, Amber Habib, Shashidhar Jagadeesan*
13:05 to 14:00	Lunch
14:00 to 14:25	*Rakhi Banerjee*: Mathematics education research in India: issues and challenges
14:25 to 14:50	*S. G. Dani*: To be announced
14:50 to 15:30	Concluding session
15:30	Tea

Conference Committees

National Program Committee

Anita Rampal, *University of Delhi*
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Local Organizing Committee (HBCSE, Mumbai)

Aaloka Kanhere
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