OPTIMIZED METHOD FOR COMPRESSIVE SENSING IN MOBILE ENVIRONMENT

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Abstract—**Compressive sensing (CS) is a novel method for channel estimation. The recently introduced principle and the methodology of compressed sensing allow the efficient reconstruction of sparse signals of a very limited number of measurements. CS has gained a fast growing interest in applied mathematics. We consider the channel estimation in mobile environment using different methods. We identified an optimized method for compressive sensing in a mobile environment after an investigation of Orthogonal Matching Pursuit (OMP) and Delay-Doppler sparsity with reduced pilots for higher spectral efficiency. We demonstrated simulation results for 4- QAM and 16- QAM with the parameters of Least Square Estimation (LSE) and CS. Our simulation results show that the Delay-Doppler Sparsity achieved good spectral efficiency along with less probability of error.**

Keywords— Compressive sensing, OFDM, Delay-Doppler sparsity, Orthogonal Matching Pursuit

I. INTRODUCTION

In a wireless environment, signal refracts diffracts and scatters from nearby objects and hence at the receiver, different signals which are attenuated, delayed and phase shifted are received. The channel state information (CSI) has an effect on overall system performance. The channel estimation is very important to estimate the parameters of channel model which is abstracted from the real time multipath propagation. OFDM is used in various digital services such as 4G, DVB, DSL, Internet access, DAB etc.

In [1] CS is applied to pilot based estimation in a highly mobile environment and channel estimation exploits a channel Delay-Doppler sparsity to reduce the number of pilots. In [2] the optimization considers the character of the channel and uses the delay of the channel to optimize the algorithm of OMP.

Conventional approaches to sampling signals or images follow Shannon's Sampling theorem. This principle requires all observed signals for reconstruction of the transmitted signal. Newly developed compressive sensing requires fewer samples to reconstruct the signal as compared to Nyquist sampling theorem, if the signal is compressed or sparse in some domain. The method of CS can reduce the dimensions of the observation matrix and the use of this reduced matrix for the reconstruction of the original signal with high probability. Sparsity and incoherence are the basic principles behind compressive sensing [3]. When compared with conventional methods of channel estimation, Compressive sensing can reach the same performance with fewer pilot signals to represent same signals.

There are various methods to reduce the number of pilots using compressive sensing technique. In this paper we compared Delay-Doppler sparsity and orthogonal matching pursuit, to decide optimized method for guaranteed recovery of the signal.

Delay-Doppler Sparsity: - From **Delay**-Doppler Sparsity we can calculate energy bounds. These bounds allow us to choose N_A where N_A is the number of non-zero samples that guarantee prescribed approximation quality.

Orthogonal Matching Pursuit: - It is a sparse approximation method which involves finding the basis of matching projections of multidimensional data over a complete dictionary.

This paper is organized as follows: - Section II describes some basic facts about compressive sensing. Section III describes multicarrier system model channel. Section IV describes Delay-Doppler sparsity. In section V, OMP algorithm is reviewed. Comparison and simulation results are given in Section VI and Section VII gives concluding remarks.

II. REVIEW OF COMPRESSIVE SENSING

Compressive sensing or CS is a novel sampling/ sensing paradigm that does not follow common wisdom in the data acquisition. CS theory shows that one can recover certain signal and images from fewer samples / measurements.

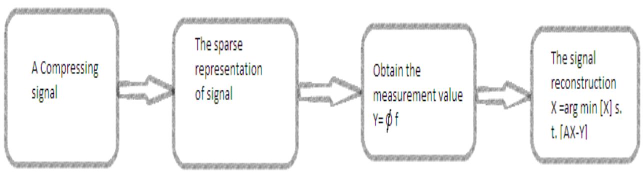

Fig. 1 The framework for compressed sensing

The sampling of a signal is a conversion of analog signal to digital information. The requirement of more information causes large pressure on the sampling of a signal. The transport of signal and storage of the signal also burdens the system. Compressive sensing provides a solution to mitigate this pressure.

CS relies on two principles, Sparsity which recovers the original signal of interest and incoherence which recovers the sensing modality.

Sparsity: - Sparsity implies that continuous time signal contains information much smaller than suggested by its bandwidth. Hence CS exploits the facts that many natural signals are compressible when they are represented with proper basis. CS provides a constructive way in exploiting this sparsity to reduce the number of pilots and hence increase spectral efficiency.

Incoherence: - We know that, time and frequency domain signals are dual incoherent. This expresses the idea that objects having sparse representation must be spread out in the domain in which they are acquired e.g. direct or spike signal in the time domain has a wide spectrum in frequency domain. Incoherence represents sampling waveform in a very dense form of Ψ where Ψ is the matrix for transformed domain . CS is a very simple and efficient signal acquisition protocol which samples the signal in independent fashion at a low rate and later uses computational power for reconstruction. As they are relatively few in number, these samples appear to be an incomplete set of measurements when compared to the set of samples using Nyquist criteria, but they are sufficient for reconstruction of the original signal.

III **MULTICARRIER SYSTEM MODEL**

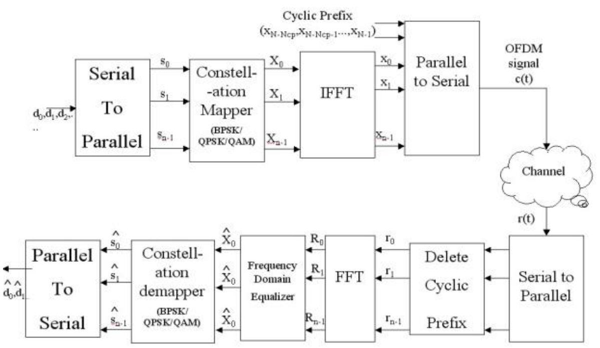

Fig.2: Base-band OFDM system

Above figure shows Orthogonal Frequency Division Multiplexing (OFDM) model for the multipath wireless channel. OFDM system can be characterized as a linear time-variant system consisting of propagation delay and Doppler Effect.

When we consider a single OFDM symbol, it can be represented as

$$X(n) = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} X(k)\, e^{\frac{j2\pi kn}{N}} \ldots\ldots\ldots\ldots(1)$$

Where X(n) are transmitted symbols and

X(k) are FFT symbols of X(n).

assuming k is greater than cyclic prefix and k is less than (N-1), where N is the number of subcarriers. Here we consider the multipath channel consisting of P propagation paths and these propagation paths have delays τ_p [2]. This multipath channel can be expressed as,

$$h(t,\tau) = \sum_{p=1}^{p} n_p\, \delta(\tau - \tau_p) \ldots\ldots\ldots\ldots(2)$$

Here n_p is the attenuation of the initial phase of path P and

τ_p is a delay of channel. In discrete case, the equation (2) can be given as

$$h(n,m) = \sum_{p=1}^{p} n_p\ \delta\left(\pi\left(m - {}^{\tau_p}/_{T_s}\right)\right) \ldots\ldots\ldots\ldots(3)$$

When signal passes through the AWGN channel continuous time signal received at the end of the multipath channel is given by,

$$r(t) = \int_{-\infty}^{\infty} h(t,\tau)s(t-\tau)d\tau + z(t) \ldots\ldots\ldots\ldots(4)$$

Where S(t)=input signal

h (t, τ)= Channel's impulse response

z (t) = Channel noise

In discrete time, the channel output is given as

$$r(n) = \sum_{p=1}^{p} x[n.m - p]h[n.,m] + z[n] \ldots\ldots\ldots(5)$$

Where z(n) = Discrete time noise, it is zero mean white complex Gaussian Noise.

Here we modeled system channel as a multichannel modulator and demodulator. Using above equations and neglecting noise in a highly mobile environment

$$x_{(l,k)} = H_{l,k}a_{l,k} + z_{l,k}$$

for l = 0,…….,L-1 and K = 0,………K-1. Here $z_{l,k} = \langle z, \Upsilon_{l,k}\rangle$ are the system channel coefficients. $H_{l,k}$ can easily be expressed in terms of h[n,m] and ϒ[n].

We will need a "Delay–Doppler domain expression" of the channel coefficients $H_{l,k}$. Let us assume that the received pulse ϒ[n] is zero outside [0, L_Υ]. To compute $x_{l,k}$ we have

$$x_{(l,k)} = < r, \Upsilon_{l,k} > = \sum_{n=-\infty}^{\infty} r[n]\Upsilon^{*}l,k[n] \ldots\ldots\ldots\ldots (6)$$

for l=0, ….,L-1, r[n] must be known for n = 0, ….,Nr-1 where Nr = (L-1)N + L_{Υ} +1. In this interval we can express r[n] as a discrete Delay-Doppler spreading function as given in eq.(5) and also as continuous time signal as given in eq. (4). We obtained the system channel relation $H_{l,k}$ expressed as

$$H_{(l,k)} = \sum_{m=-\infty}^{\infty} \sum_{i=0}^{Nr-1} F[m,i] e^{-j2\pi(\frac{km}{K} - \frac{Nl_i}{Nr})} \ldots\ldots\ldots\ldots (7)$$

Where

$$F[m,i] = s_h[m,i] A^{*}_{\Upsilon,g}(m, \frac{i}{Nr})$$

Where the ambiguity function is given by

$$A^{*}_{\Upsilon g}(m, \frac{i}{Nr})$$

Using the approximation Nr = LN, we can write Delay-Doppler sparsity as given in eq. (8) which is a two dimensional discrete fourier transform

$$H_{(l,k)} = \sum_{m=0}^{K-1} \sum_{i=0}^{L-1} \tilde{F}[m,i] e^{-j2\pi(\frac{km}{K} - \frac{l_i}{L})} \ldots\ldots\ldots\ldots (8)$$

above equation with "pre-aliased" version of F[m,i] can be given as

$$\tilde{F}[m,i] = \sum_{q=0}^{N-1} F[m, i+qL], \quad i = 0, \ldots, L-1 \ldots\ldots\ldots\ldots (9)$$

IV DELAY-DOPPLER SPARSITY

Delay Doppler Sparsity is related by ambiguity function which is a two-dimensional function of time delay and Doppler frequency X(τ,f). It shows the distortion of return pulse due to receiver match filter which is the effect of Doppler Shift. Ambiguity function is suitable to represent propagation delay and Doppler relationship of wide band signals. For a given complex baseband pulse, S(t) is the narrow band ambiguity function is given by

$$x(\tau, f) = \int_{-\infty}^{\infty} s(t) s^{*}(t-\tau) e^{i2\pi f t} dt \ldots\ldots\ldots\ldots (10)$$

Where

*is complex conjugate

i is an imaginary unit

When there is no Doppler shift i.e. f = 0 above equation reduces to auto-correlation of S(t)

In [1], by using above ambiguity function, Delay-Doppler sparsity equation reduces the channel response for mobile radio channel and it is given as,

$$h(t,\tau) = \sum_{p=1}^{P} n_p\, \delta(\tau - \tau_p) e^{j2\pi v_p t} \ldots\ldots\ldots\ldots (11)$$

Where v_p is frequency shift for P propagation paths

n_p is attenuation

Above equation gives the relation between τ_p Doppler frequency shift v_p

In discrete time, channel impulse response can be given as

$$h[n,m] = \sum_{p=1}^{P} n_p\, e^{j2\pi v_p n T s}\ \operatorname{sinc}\left(\pi\left(m - \frac{\tau_p}{T_s}\right)\right) \ldots\ldots\ldots\ldots (12)$$

Finally Delay-Doppler spreading function is obtained as

$$S_n[m,i] = \sum_{p=1}^{P} n_p\, e^{j\pi\left(v_p T_s - \frac{i}{Nr}\right)(Nr-1)} \wedge_p [m,i] \ldots\ldots\ldots\ldots (13)$$

Where

$$\wedge_p[m,i] = sinc\left(\pi\left(m - \frac{\tau_p}{T_s}\right)\right) dir_{Nr}\left(\pi\left(i - v_p T_s Nr\right)\right) \ldots\ldots\ldots (14)$$

Here we will consider $\wedge_p$[m,i] as $N_{\wedge}$ - sparse, with an approximately chosen number $N_{\wedge}$ of non zero samples , The energy bound in [1] allows us, $N_{\wedge}$ such that prescribed approximation quality can be guaranteed.

V ORTHOGONAL MATCHING PURSUIT ALGORITHM

Matching Pursuit: - It is a greedy iterative algorithm for approximatively solving l_0 pseudo- norm problems where l_0 norm can be given as

$$\|\alpha\|_0 = \{i: \alpha_i \neq 0, i = 1, \ldots, p\}$$

is a pseudo-norm l_0 which counts the number of non zero component of α.

Matching pursuit works by finding a basis vector in D that maximizes the correlation with the residual. Again compute the residual & coefficients. We project the residual on all atoms in the dictionary. The drawback of matching pursuit algorithm is that it picks the same atoms multiple times. The OMP gives the solution for this problem. OMP is modified version of MP except that an atom picked at once does not pick again. For this purpose, algorithm maintains an active set of atoms which are already picked. For every iteration a new atom gets added with the existing atom.

OMP Algorithm: In OMP the residual is always orthogonal to the atom already selected. This means that the same atom can never selected twice and result in convergence for a d-dimensional vector after at most d steps. By using Gram-Schmidt procedure to find an orthonormal set of atoms.

The OMP algorithm is

1: Denote your signal by f, initialize the residual $R^0 f = f$.

2: Select the atom that maximizes the absolute value of the inner product with $R^0 f = f$. Denote that atom by ϕ_p.

3: Form a matrix ϕ with previously selected atoms as the columns. Define orthogonal projection operator onto the span of columns ϕ.

$P = \phi (\phi * \phi)^{-I} \phi *$

Where ϕ= dictionary of atoms as NxM matrix with M> N

4: Apply the orthogonal projection operator to the residual.

5: Update the residual

$R^{m+1} f = (I-P) R^m f$

Where I is the identity matrix.

Main attribute of OMP algorithm is its stopping structure which depends upon the characteristics of noise.

In noiseless case, $R^{m+1} f = 0$ and iteration for the algorithm will be stopped.

For different values of noise structure different values of $R^{m+1} f$ will be considered.

VI. COMPARISON BETWEEN OMP ALGORITHM FOR MOBILE ENVIRONMENT AND DELAY DOPPLER SPARSITY

A.DELAY DOPPLER SPARSITY

In [1] following parameters are considered for simulation:

1: No. of subcarriers K= 2048

2: Cyclic Prefix Length (N-K) =512 where N= No. of samples

3: N=2560

4: Modulation method=4 QAM

B. OMP METHOD

In [2] following parameters are considered for simulation:

1: No. of subcarriers K= 1024

2: CP length (N-K) =256

3: N=1280

4: Modulation Method= 16 QAM

Using above parameters, for known channel estimation the result are shown below,

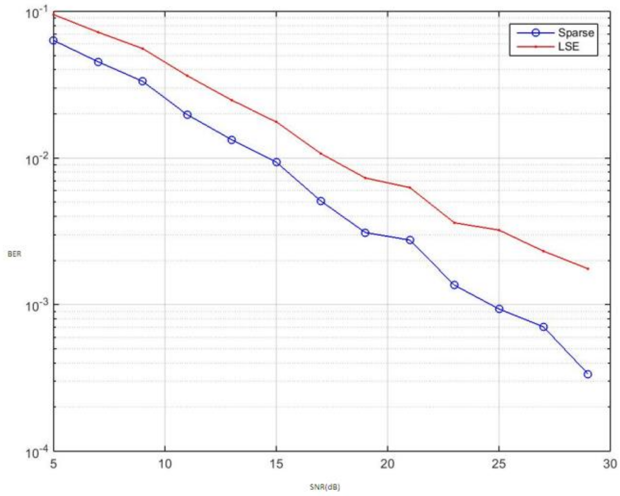

Fig3: Bit Error Rate by using Delay-Doppler Sparsity with K=2048

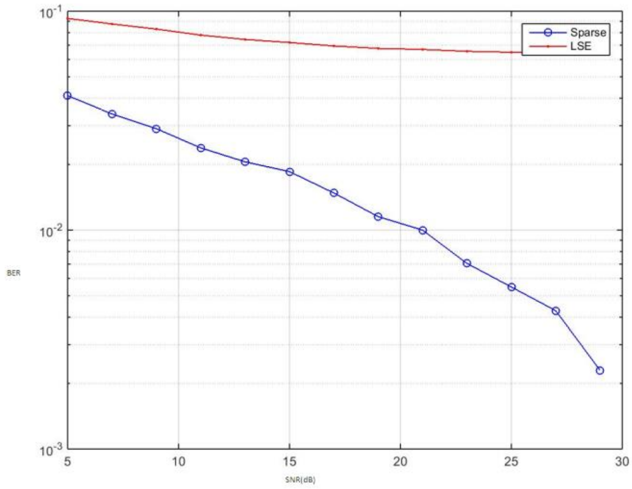

Fig.4: Bit Error Rate Bit Error Rate by using Delay-Doppler Sparsity with K=1024

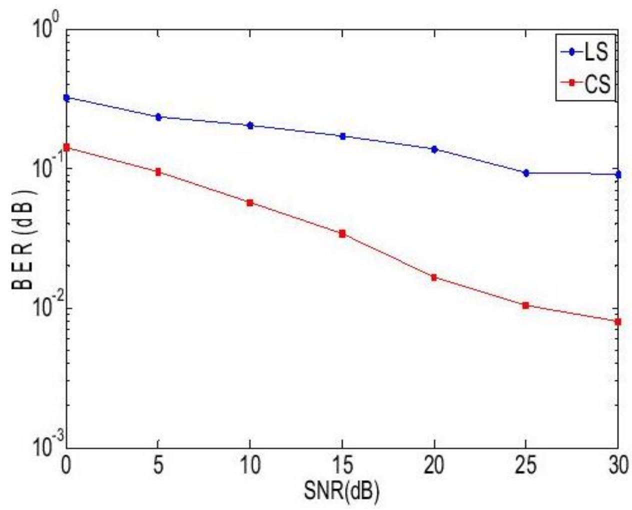

Fig.5: Bit error rate by using OMP method

1. For the unknown channel estimation, the pilots are required and the energy required for transmission of signal with pilot is more.
2. Neglecting above fact, in 4-QAM constellation the signal energy Es=2 and location of bits are also far from each other, so the probability of error is less.
3. In 16 QAM constellations the signal energy is 10 and the location of bits are also nearer to Euclidean distance, so the probability of error is more.
4. When probability of error is more, there will be constraint on the reduction of pilots.

Optimum pilot signal design is a separate issue which enhance the complexity of modulation .Pilot symbols does not carry any information about the data, hence the time spend on the sending pilot symbol is a time missed for transmitting information .For transmission of pilots the power is required which is taken away from the data symbols. The location of these pilots in the data stream also affect the system performance in terms of reliable transmission rate, bit error rate or mean square error of the estimator.

The pilot symbols are needed throughout the transmission so that new user can acquire the channel state information (CSI) and gain synchronization. This implies that the fixed percentage of pilots should be embedded in the data stream.

VII. CONCLUSION

Based on the recently introduced methods for compressed channel estimation, we studied the Delay- Doppler sparsity and OMP .Our results demonstrate that the Delay-Doppler sparsity is superior to the OMP algorithm under the condition of reduced pilots. We observed that while maintaining the same data rate, we can achieve less probability of error and reduced hardware complexity with Delay-Doppler sparse channel estimation.

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